The Simple Linear Regression Model

From Scatter Plot to Population Model

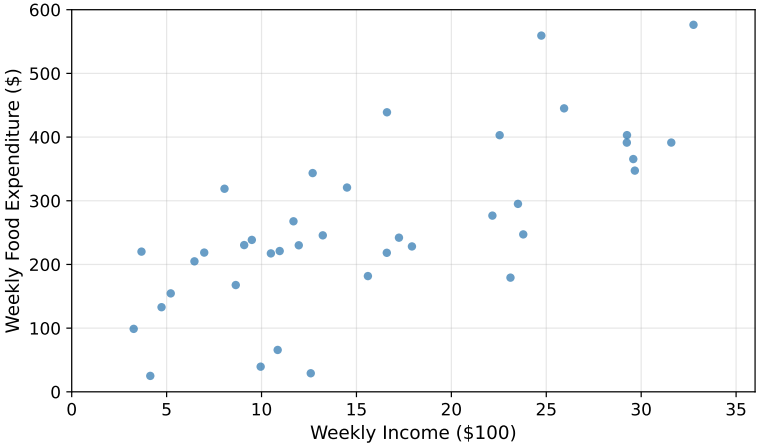
Jake Anderson

March 21, 2026

Outline

Back to the Food Expenditure Example

In Lecture 1 you saw a scatter plot of **weekly food expenditure** vs. **weekly household income** for 40 households from southern Australia.



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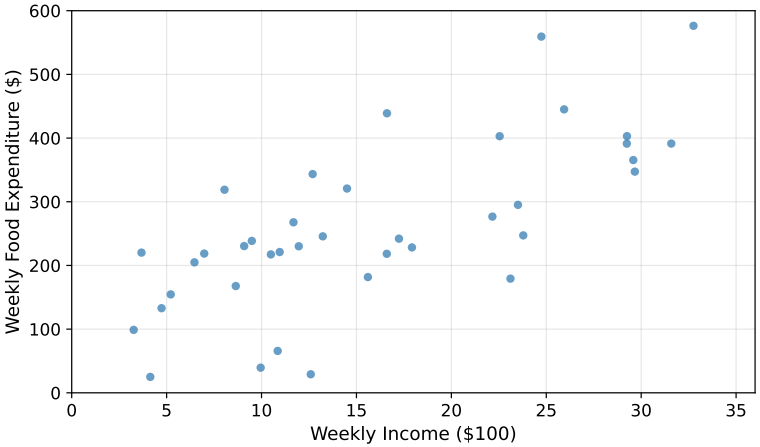

A positive relationship is visible. But can we be more precise? What is the *average* food expenditure

What We Want to Know

Two concrete questions:

1. If weekly income goes up by \$100, how much does *average* weekly food expenditure rise?
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We previewed the equation $y = \beta_1 + \beta_2 x + e$ in Lecture 1. Now let's build it from scratch and understand every piece.

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A Deterministic Starting Point

Imagine every household follows a fixed spending rule: \$83 base plus \$10 for every \$100 of weekly income goes to food.

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Let's check this against the data.

The Deterministic Model Falls Apart

Using $y = 83 + 10x$, consider two households:

- **Household A:** income $x = 20$. Model predicts $y = 283$. Actual spending: \$350.
- **Household B:** income $x = 15$. Model predicts $y = 233$. Actual spending: \$120.

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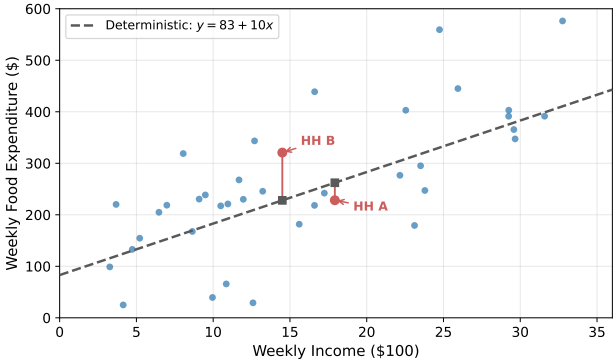

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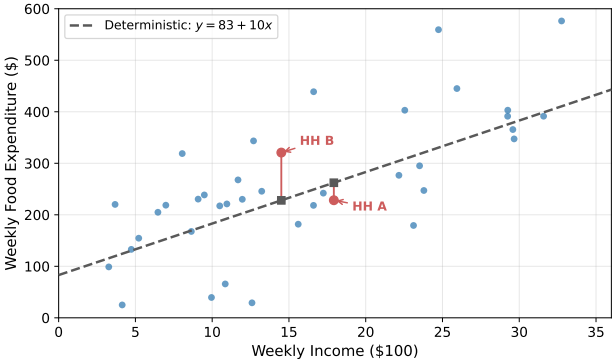

$\Longrightarrow$ At the same income, different households spend very different amounts. What is the model

Adding the Error Term

What explains the variation in food expenditure *beyond* income?

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- β_2: slope (change in *expected* food expenditure per \$100 increase in income)
- e: random error (everything else affecting y)

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β_1 and β_2 are unknown **population parameters**. We never observe them directly.

The Model for Each Observation

We have $N = 40$ households, randomly sampled. Each one satisfies the same behavioral rule:

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This equation is the **data generating process (DGP)**: it describes how the observable data arise from the unknown parameters plus randomness.

Decomposing y_i

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The systematic component will turn out to be the *conditional mean* $\mathrm{E}(y_i \mid x_i)$, once we impose an assumption on e_i.

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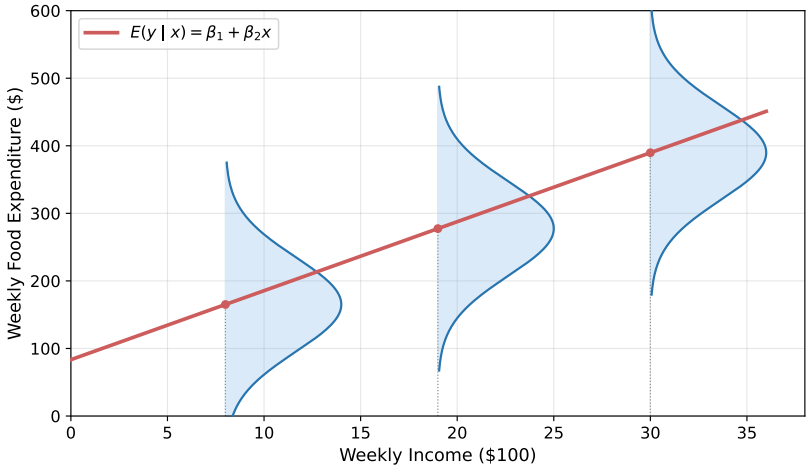
What Does “Average Food Expenditure at a Given Income” Mean?

At any income level x, there is a **distribution** of food expenditure values across households. Some spend more, some less.



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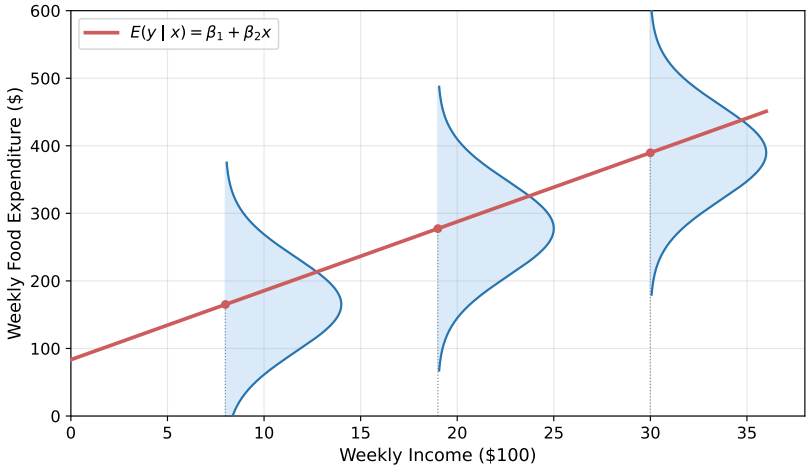

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$$\begin{aligned} E(y_i \mid x_i) &= E(\beta_1 + \beta_2 x_i + e_i \mid x_i) \\ &= \beta_1 + \beta_2 x_i + \underbrace{E(e_i \mid x_i)}_{=0} \end{aligned}$$

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A one-unit increase in x (here, \$100 in weekly income) changes expected food expenditure by β_2 dollars, holding all else constant.

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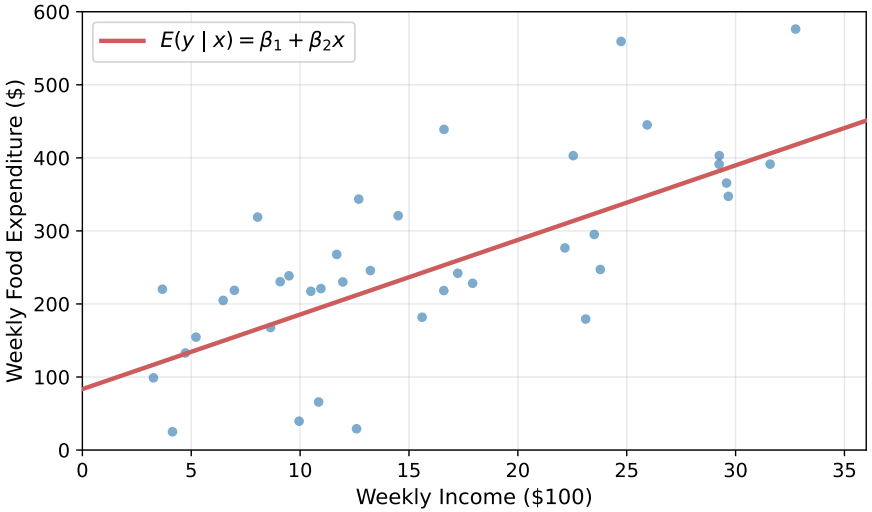
$\Longrightarrow$ The intercept is often just a mathematical anchor for the line, not an economically meaningful quantity.

The Regression Line Through the Data



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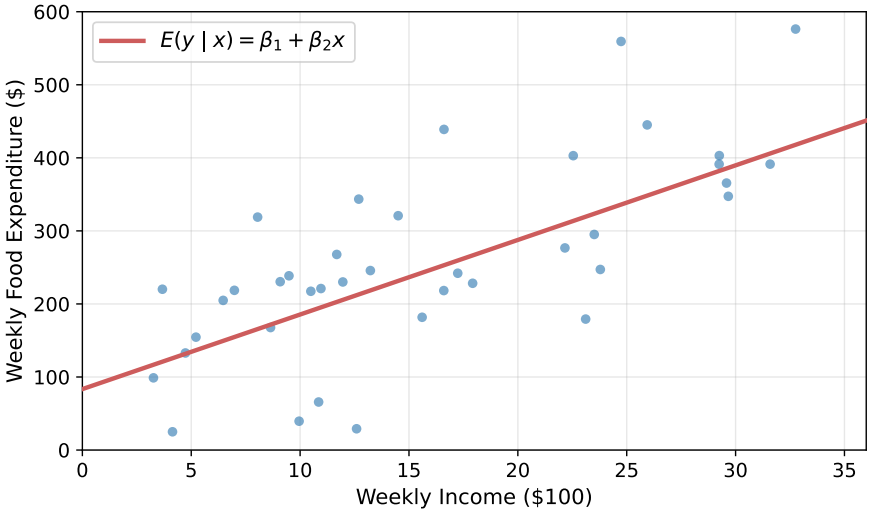

Each point y_i is the sum of the predicted value on the line ($E(y_i \mid x_i) = \beta_1 + \beta_2 x_i$) plus the error ($e_i$)

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Why Do We Need Assumptions?

The model $y_i = \beta_1 + \beta_2 x_i + e_i$ is just notation. Without assumptions about e_i and x_i, we cannot:

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For each assumption, we will ask: **what goes wrong if it fails?**

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If the true relationship is, say, $E(y \mid x) = \beta_1 + \beta_2 x^2$, then fitting a straight line produces systematically wrong predictions. The errors e_i would have a pattern (positive at low and high x, negative in the middle), which violates later assumptions too.

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$\Longrightarrow$ We will revisit nonlinear models (quadratic, log-linear) in a later lecture.

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Without SR2, OLS estimates are **biased**: on average, the estimated slope does not equal the true slope. If x is correlated with the error, this bias does not disappear as the sample grows; OLS is also **inconsistent**.

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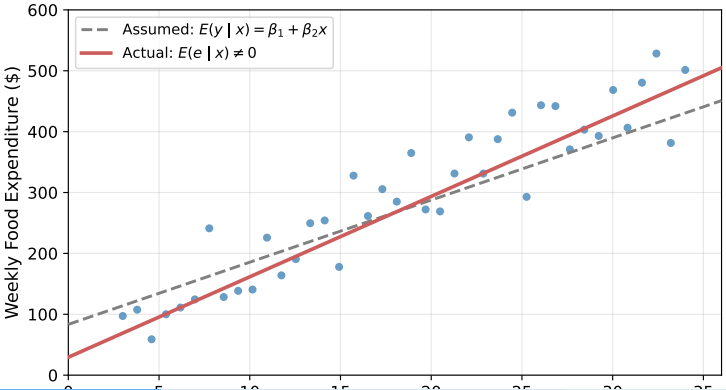

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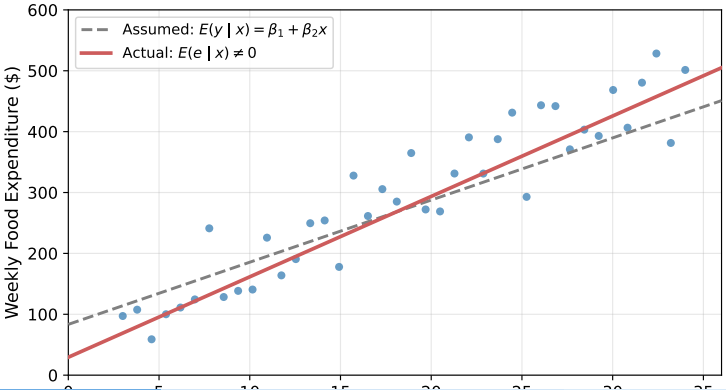

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- But is this *correlated with income*? Do higher-income households systematically have more expensive tastes, beyond what income itself predicts?

$\implies$ For this model, $\mathrm{E}(e_i \mid x_i) = 0$ is at least arguable. Whether it holds is ultimately an empirical and theoretical judgment, not something we can prove from the data.

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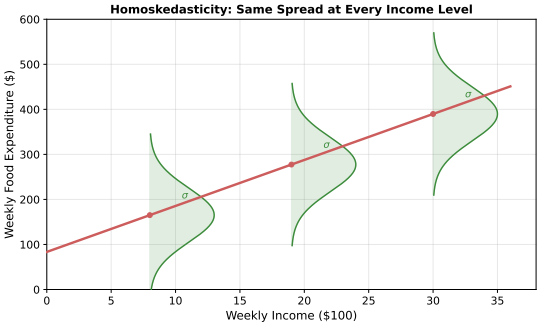
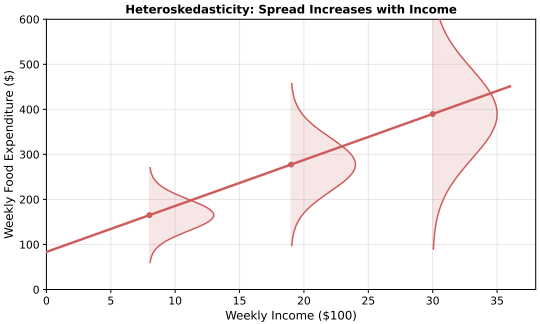
Since $\text{Var}(y_i \mid x_i) = \text{Var}(e_i \mid x_i)$, this also means $\text{Var}(y_i \mid x_i) = \sigma^2$.

SR3: Homoskedasticity vs. Heteroskedasticity





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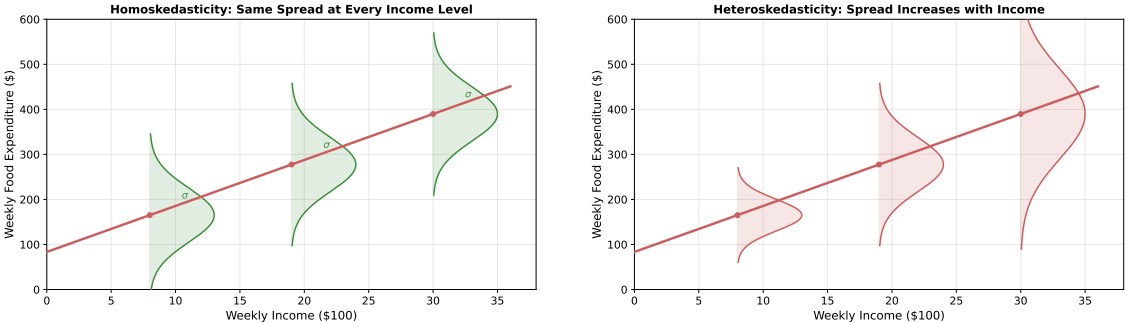
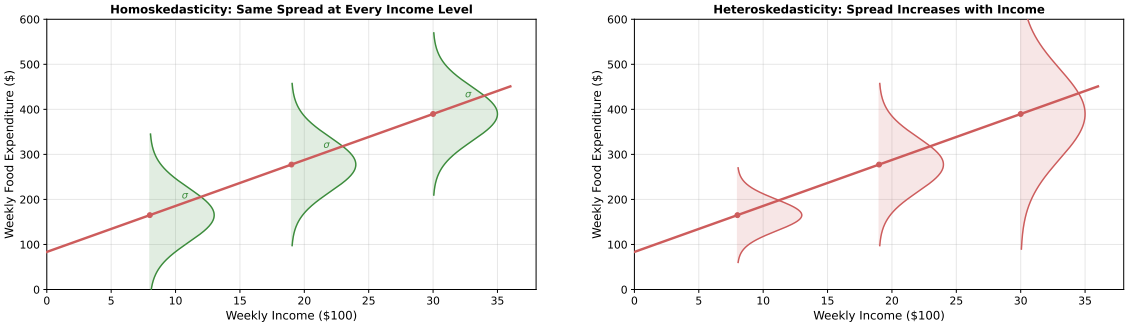


Left: Same spread at every income level (SR3 holds).
Right: Higher-income households have more variable food spending (SR3 fails). This is **heteroskedasticity**.

SR3: Homoskedasticity vs. Heteroskedasticity



Left: Same spread at every income level (SR3 holds).
Right: Higher-income households have more variable food spending (SR3 fails). This is **heteroskedasticity**.

$\Longrightarrow$ If SR3 fails, OLS estimates are still unbiased, but their **standard errors are wrong**. Confidence intervals and hypothesis tests become unreliable.

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$\implies$ If SR4 fails, the consequences are similar to SR3: OLS is still unbiased, but standard errors are wrong.

SR5: Variation in x

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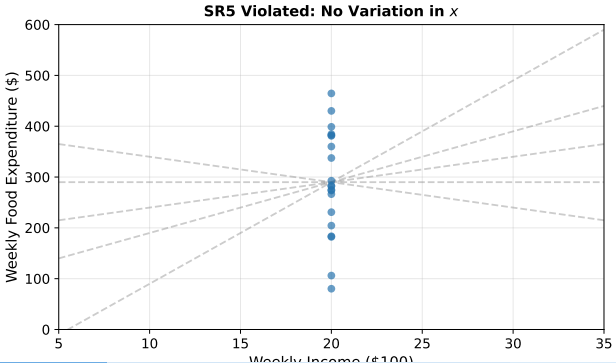

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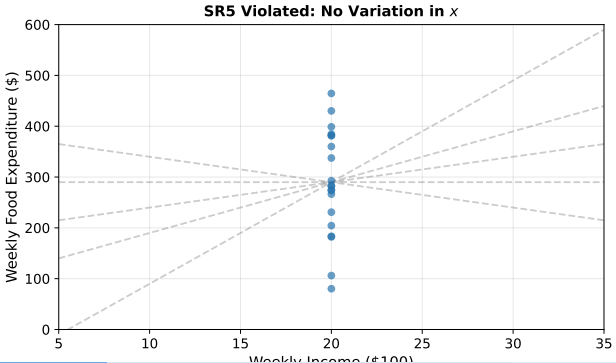

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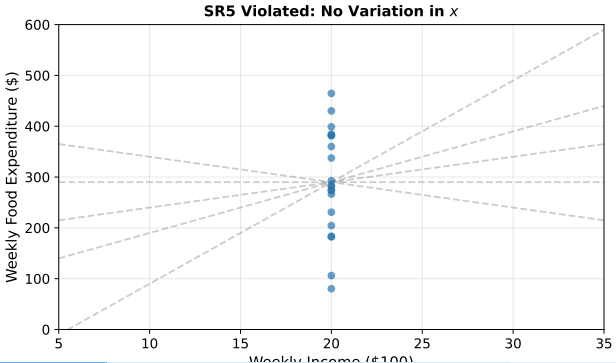

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Why optional? If N is large enough, the CLT also makes the *estimators* b_1 and b_2 approximately normal, even when the e_i are not. SR6 is most useful in small samples.

All Six Assumptions at a Glance

Label	Name	Statement
SR1	Linear model	$y_i = \beta_1 + \beta_2 x_i + e_i$
SR2	Zero conditional mean	$\mathrm{E}(e_i \mid x_i) = 0$
SR3	Homoskedasticity	$\mathrm{Var}(e_i \mid x_i) = \sigma^2$
SR4	Uncorrelated errors	$\mathrm{Cov}(e_i, e_j \mid x_i, x_j) = 0$ for $i \neq j$
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SR1–SR5 are needed for OLS to be the Best Linear Unbiased Estimator (Gauss–Markov theorem, coming in a future lecture). SR6 adds exact distributional results for small samples.

What Goes Wrong: A Summary

Violated	Example	Consequence
SR1	True relationship is quadratic	Systematic prediction errors
SR2	Ability correlated with education	OLS is **biased**
SR3	Rich households have more variable spending	Standard errors are wrong
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$\Longrightarrow$ SR2 is the most consequential: it determines whether OLS gives unbiased answers. The rest of the course will develop tools for detecting and correcting these violations.

Outline

Two Vocabularies

	Population (unknown)	Sample (computed)
Intercept	β_1	b_1
Slope	β_2	b_2
Error	e_i	$\hat{e}_i$ (residual)
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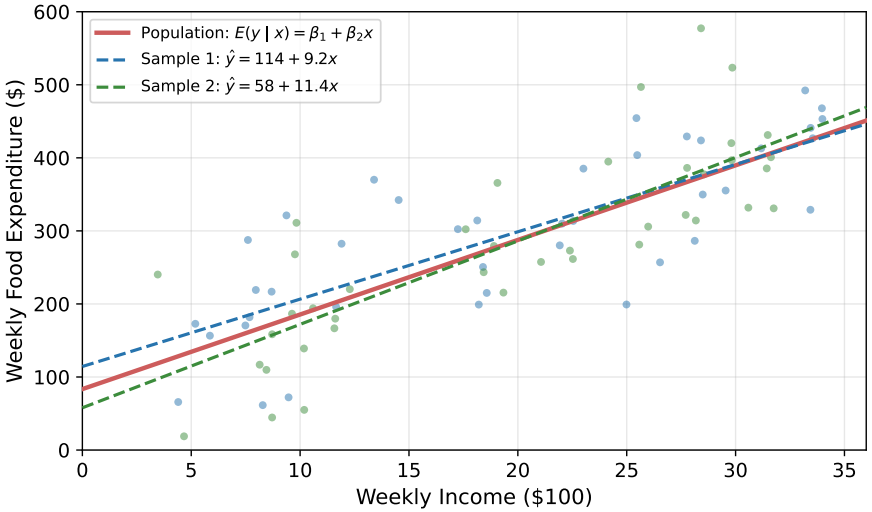
Estimators (b_1, b_2 as formulas, not numbers) are **random variables** whose properties (bias, variance) we can study.

Different Samples, Different Estimates



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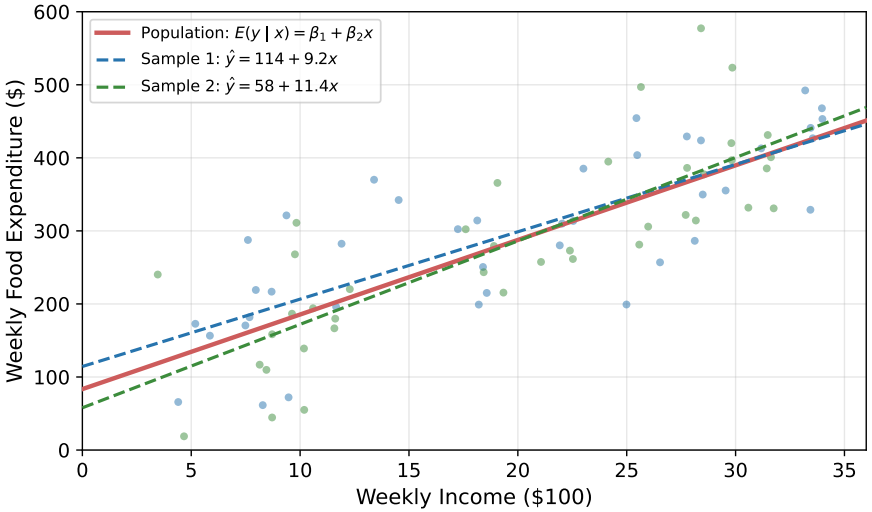

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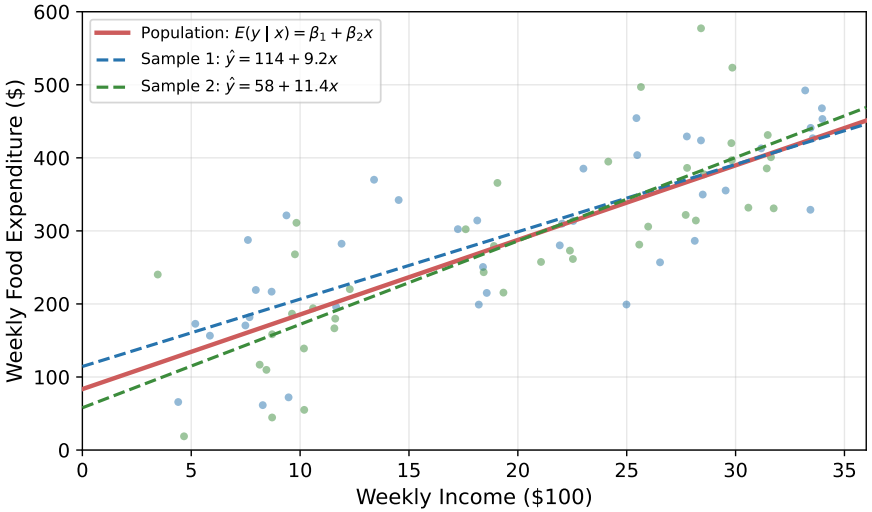

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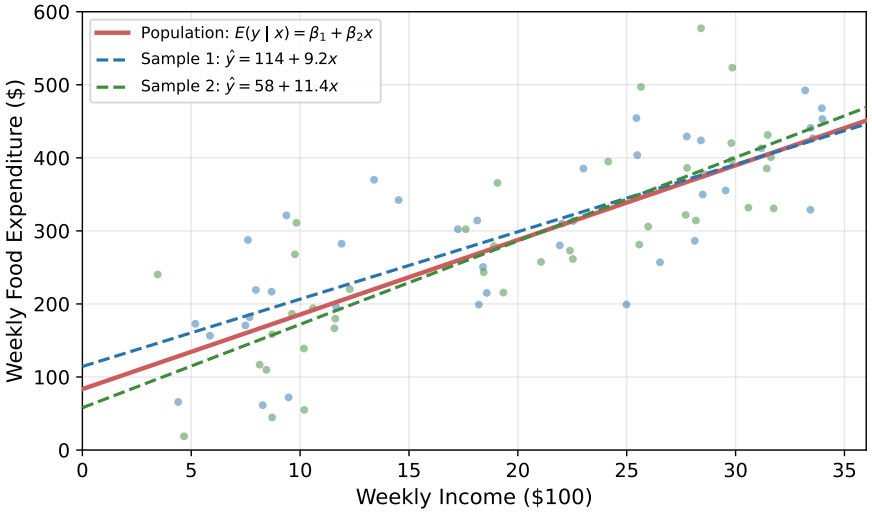

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$\Longrightarrow$ Under SR1–SR5, the OLS estimators have desirable properties that we will prove.

Thank you!
jakeanderson@g.ucla.edu