Ordinary Least Squares Estimation

How to Draw the Best Line Through a Scatter Plot

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UCLA Economics

Econ 103 – Lecture 3

Outline

The Food Expenditure Data

$N = 40$ three-person households from southern Australia.

- y_i: weekly food expenditure per person ($)
- x_i: weekly household income (in $100 units)

	Food Expenditure ($)	**Weekly Income ($100)**
Mean	283.57	19.60
Std. dev.	112.68	6.85
Min	109.71	3.69
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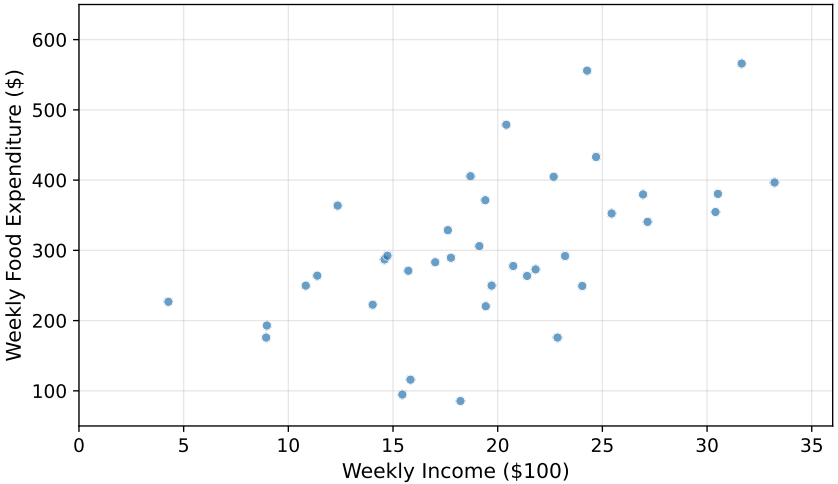
Do richer households spend more on food? Let's look at the scatter plot.

The Scatter Plot



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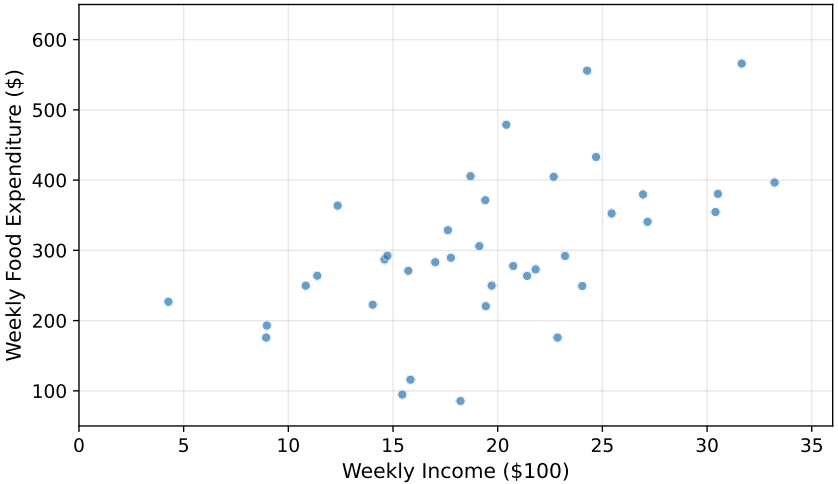
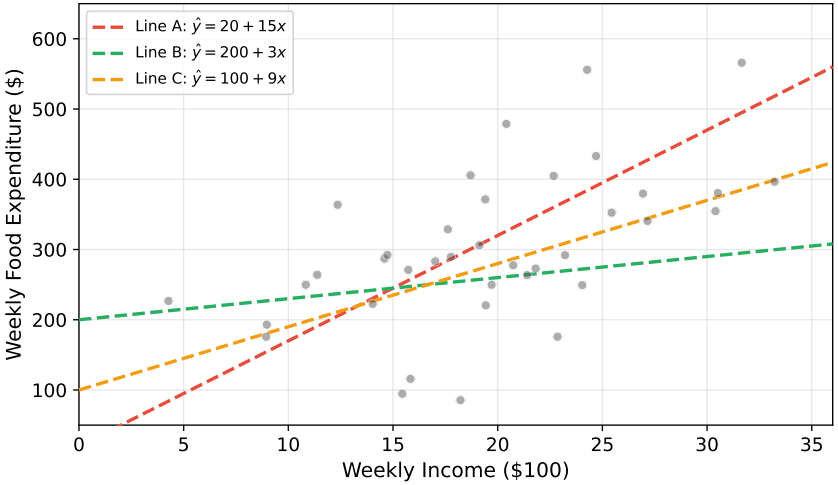


Many lines could be drawn through these points. Which one is “best”?

Eyeballing Fails



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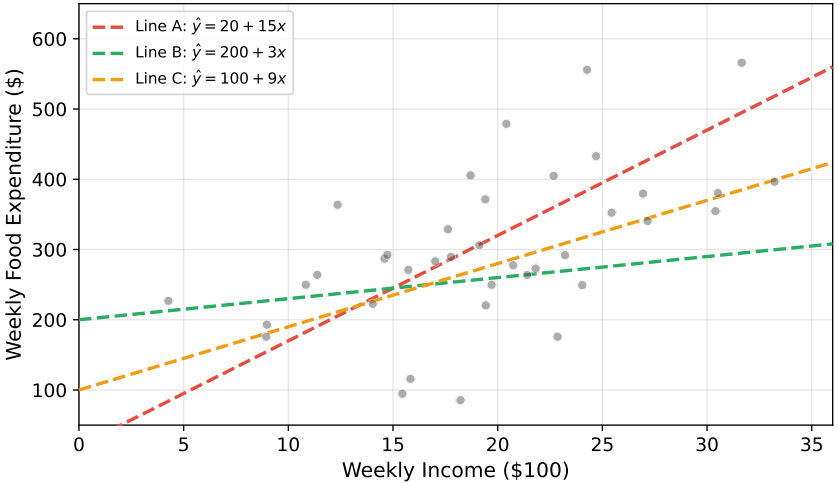

Three different people could draw three different "best" lines. We need an **objective criterion** for choosing the line.

Recap: The Simple Linear Regression Model

From Topic 5, we have the simple linear regression model:

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We need estimates b_1 and b_2 to draw a fitted line through the scatter plot.

Residuals: Measuring the Misfit

For any candidate line $\hat{y}_i = b_1 + b_2 x_i$, the **residual** for observation i is:

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Why not minimize $\sum \hat{e}_i$? Because positive and negative residuals cancel out. A terrible line through the middle could have $\sum \hat{e}_i = 0$.

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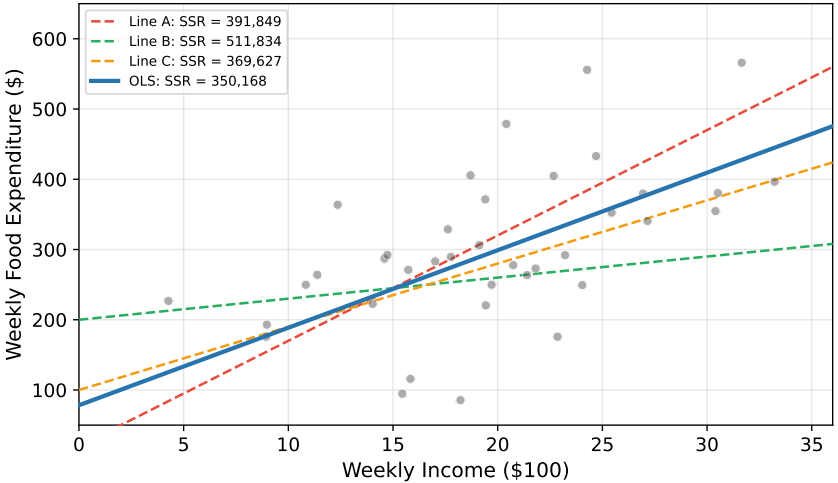
The values of b_1 and b_2 that achieve this minimum are the **Ordinary Least Squares (OLS) estimators**.

Comparing Lines by Their SSR



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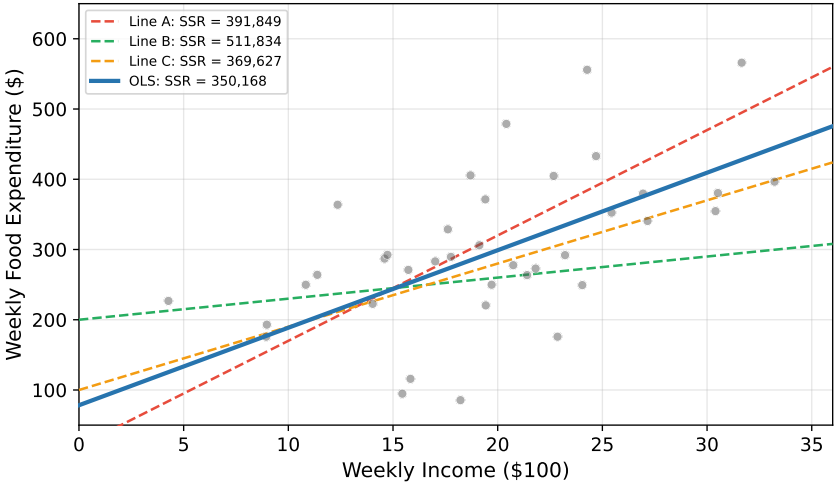

The OLS line has the smallest possible SSR. No other line can do better.

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Strategy: take partial derivatives with respect to b_1 and b_2, set them equal to zero, and solve.

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Taking the partial derivative with respect to b_1:

$$\frac{\partial S}{\partial b_1} = -2 \sum_{i=1}^{N} (y_i - b_1 - b_2 x_i) = 0$$

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Rearranging both, we get the **normal equations**:

$$Nb_1 + \left(\sum x_i\right) b_2 = \sum y_i$$

$$\left(\sum \quad\right) b_1 + \left(\sum \quad^2\right) b_2 = \sum$$

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Once we find b_2, we get b_1 for free.

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Using the summation identities (you can verify these by expanding $(x_i - \bar{x})(y_i - \bar{y})$ and distributing the sum):

$$N\sum x_i y_i - \sum x_i \sum y_i = N\sum (x_i - \bar{x})(y_i - \bar{y})$$
$$N\sum x_i^2 - (\sum x_i)^2 = N\sum (x_i - \bar{x})^2$$

we can rewrite b_2 in a more revealing form.

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$\Longrightarrow$ The slope estimate captures how y moves with x, scaled by how much x moves on its own.

The OLS Estimators: Summary

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The **fitted line** is: $\hat{y}_i = b_1 + b_2 x_i$

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Now let's use them on the food expenditure data.

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Interpretation: A \$100 increase in weekly income is associated with a \$10.21 increase in expected weekly food expenditure.

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The fitted regression line:

$$\hat{y}_i = 83.42 + 10.21\, x_i$$

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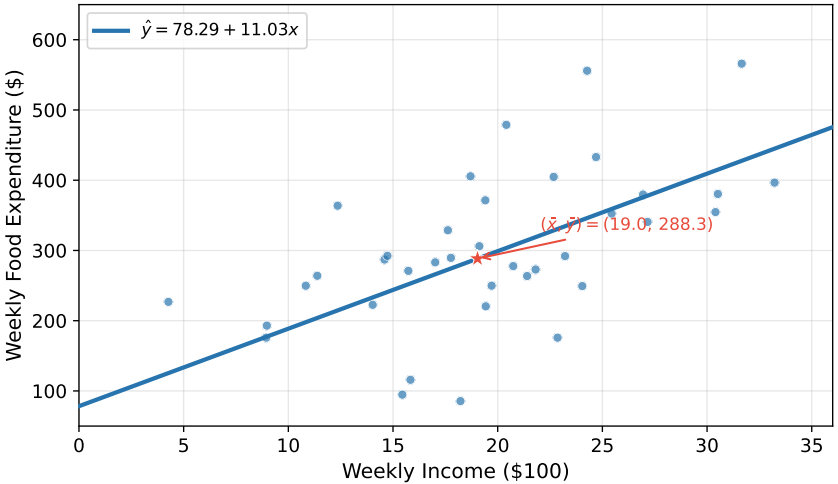
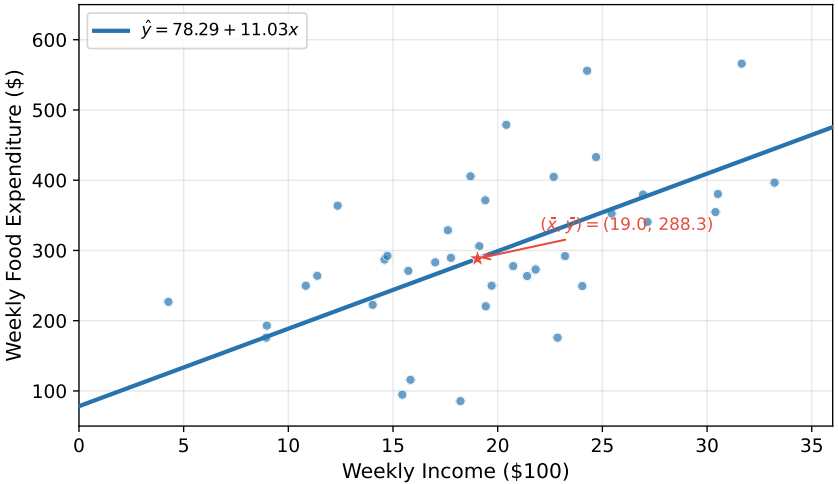


The OLS Fitted Line



The line passes through $(\bar{x}, \bar{y}) = (19.60, 283.57)$, as guaranteed by the formula $b_1 = \bar{y} - b_2\bar{x}$.

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$\Longrightarrow$ Every data point decomposes as:

$$y_i = \underbrace{\hat{y}_i}_{\text{explained}} + \underbrace{\hat{e}_i}_{\text{unexplained}}$$

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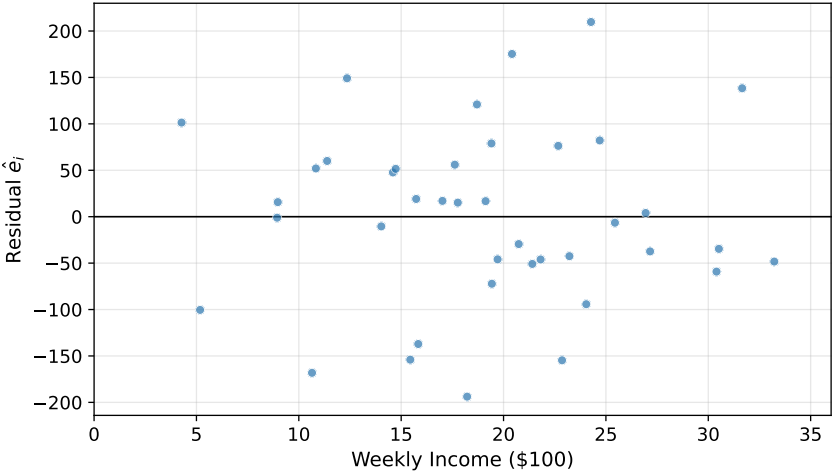
$\Longrightarrow$ The residual captures household-specific factors (tastes, family composition, etc.) that the model does not explain.

Visualizing the Residuals



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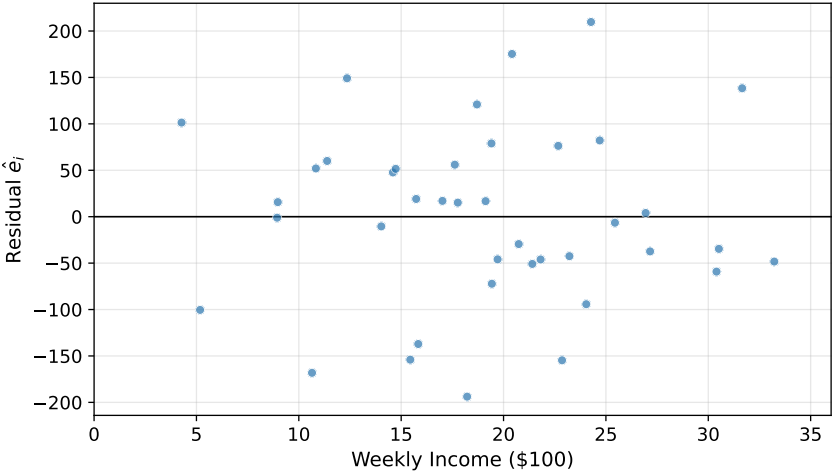

The residuals scatter around zero with no obvious pattern. If you see a pattern (e.g., a fan shape, a curve), that suggests the model is missing something.

Estimators vs. Estimates

Estimator: the formula $b_2 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$ (a **random variable**).

Estimate: the number $b_2 = 10.21$ from our particular sample (a **fixed number**).

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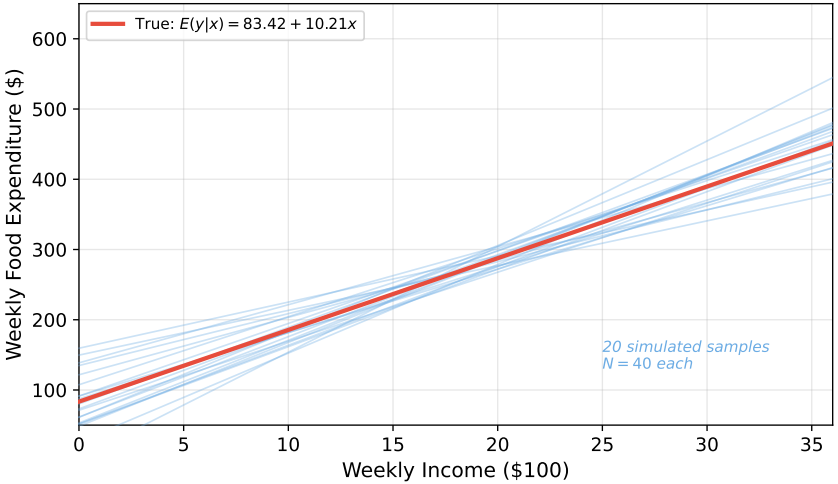
The estimator has a probability distribution, a mean, and a variance. Understanding these properties is the subject of the next few topics.

Sampling Variation: Different Samples, Different Estimates



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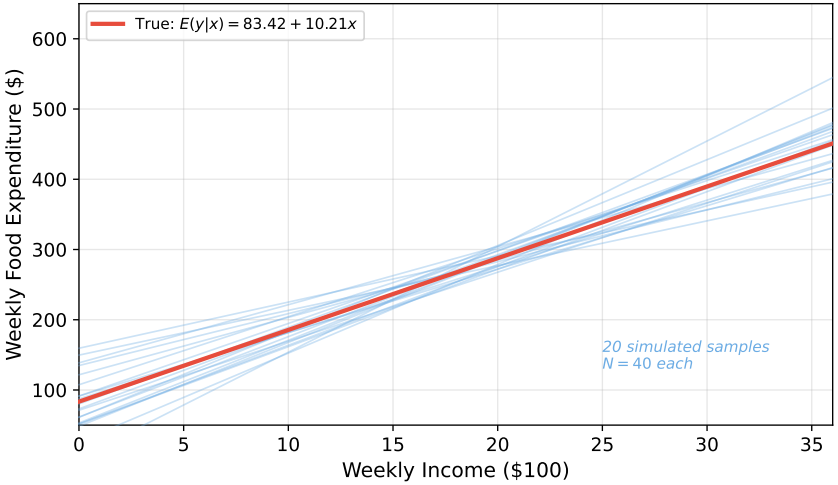

Each sample of $N = 40$ households from the same population produces a different fitted line. The slopes cluster around the true β_2 — but no single sample hits it exactly.

Ten Hypothetical Samples

The true parameter values are $\beta_1 = 83.42$ and $\beta_2 = 10.21$. If we could repeatedly draw samples of $N = 40$ from the same population:

Sample	b_1	b_2
1	93.64	8.24
2	91.62	8.90
3	126.76	6.59
4	55.98	11.23
5	87.26	9.14
6	122.55	6.80
7	91.95	9.84
8	72.48	10.50
9	90.34	8.75
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Average	96.11	8.70

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The estimates bounce around. Ten samples is too few for the average to converge to β_1 and β_2, but

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$\Longrightarrow$ So next we ask: how reliable is OLS?

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$\Longrightarrow$ Topics 7–8 address these questions using assumptions SR1–SR5 and the Gauss–Markov theorem (normality, SR6, is not needed here).

Thank you!
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