Estimating the Error Variance and Prediction

How Precise Are Our Estimates, and What Can We Predict?

Jake Anderson

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$\Longrightarrow$ Today: how to quantify how much b_2 would bounce around across repeated samples.

Outline

1. Where We Stand
2. Estimating σ^2
3. Standard Errors of b_2
4. Point Prediction
5. Elasticity at the Means
6. Summary

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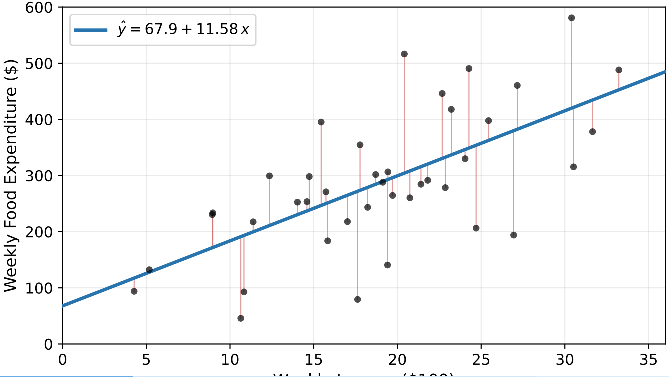

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Correction: divide by $N-2$ instead of N.

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Analogy: sample variance uses $N-1$ because estimating $\bar{x}$ uses 1 df. Here, estimating a line uses 2 df.

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Food expenditure example (Example 2.5):

$$\hat{\sigma}^2 = \frac{304{,}505.2}{40-2} = \frac{304{,}505.2}{38} = 8{,}013.29$$

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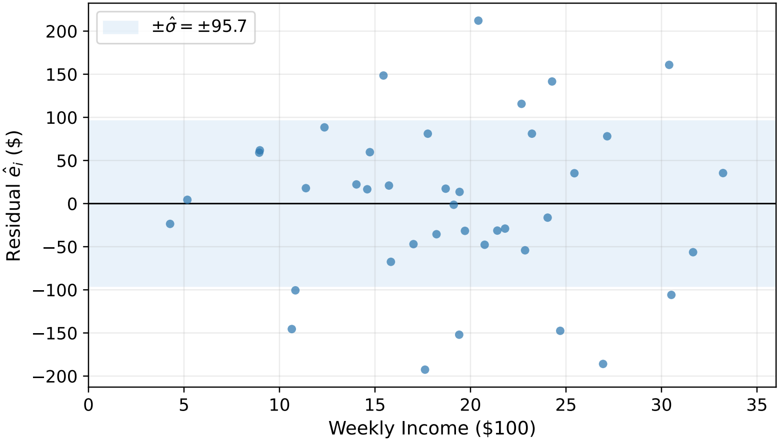
$\Longrightarrow$ Typical deviation of actual food spending from the regression line is about $89.52.

Visualizing the Residual Spread



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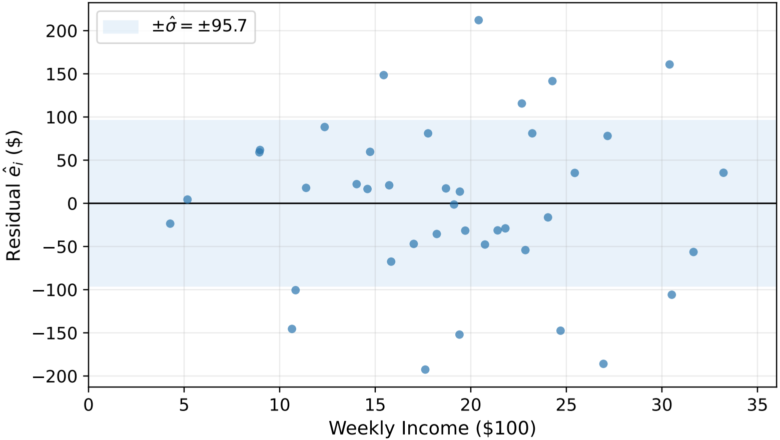

$\hat{\sigma}$ estimates the typical spread of y around the fitted line. Under homoskedasticity (SR3), this spread is the same at every income level.

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(The intercept variance $\widehat{\text{Var}}(b_1)$ has a similar formula; software computes it automatically.)

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Also: larger N increases $\sum(x_i - \bar{x})^2$, so more data improves precision.

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The numbers in parentheses below the coefficients are standard errors. This is standard notation in econometrics.

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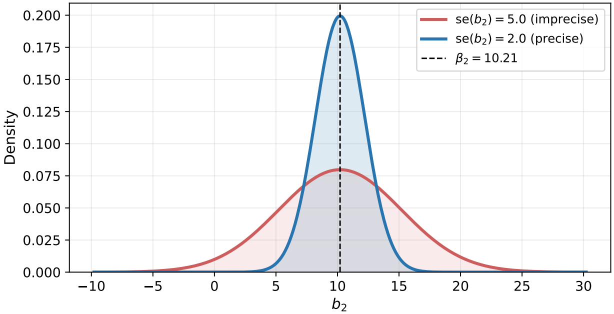

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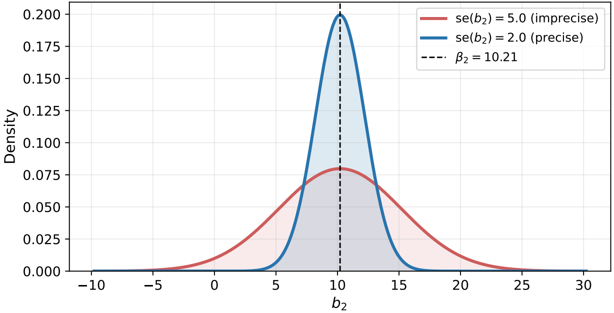

$\Longrightarrow$ Smaller $\text{se}(b_2)$ means the sampling distribution is more concentrated around β_2. Our estimates are more **reliable**.

What Makes se(b_2) Small?

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In practice, we control N (collect more data) and $\sum(x_i - \bar{x})^2$ (sample households with diverse incomes). We rarely control σ.

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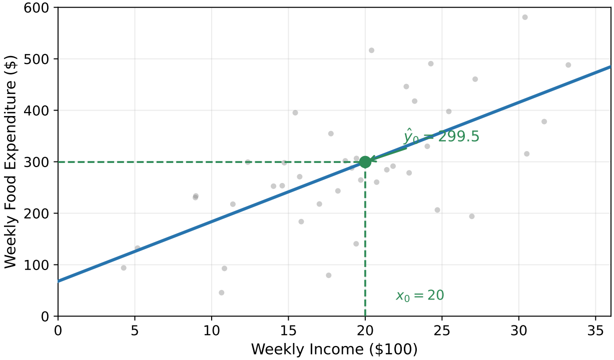

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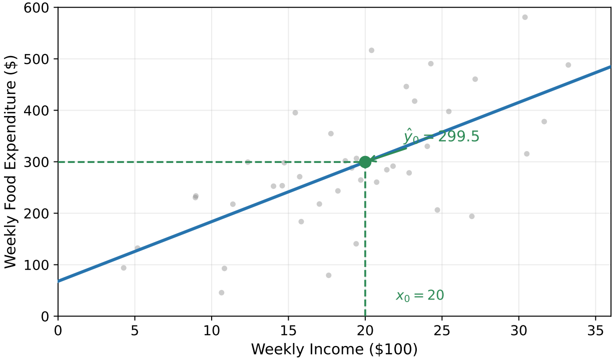

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$\Longrightarrow$ Prediction is always less precise than estimation. We formalize this in Chapter 4.

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Convention: evaluate at the sample means $(\bar{x}, \bar{y})$.

Computing the Elasticity

Estimated elasticity at the means:

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Since $\hat{\varepsilon} < 1$, food is a **necessity**: spending grows, but slower than income.

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$\Longrightarrow$ Always specify where you evaluate the elasticity ("at the means," "at $x = 25$," etc.).

Outline

1 Where We Stand

2 Estimating σ^2

3 Standard Errors of b_2

4 Point Prediction

5 Elasticity at the Means

6 Summary

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Next: now that we can report precision, what do we *do* with it? $\implies$ confidence intervals and hypothesis tests (Chapter 3).

Thank you!

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