Interval Estimation and Confidence Intervals

From Point Estimates to Ranges of Plausible Values

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Outline

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Last time we estimated the food expenditure model and reported:

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$\Longrightarrow$ A point estimate and a standard error are the ingredients. A **confidence interval** is the recipe that turns them into a range of plausible values for β_2.

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Step 1: b_2 Is Normally Distributed

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and rearrange to get an interval for β_2. But we do not know σ^2.

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Replacing the known σ with $\hat{\sigma}$ in the denominator adds estimation uncertainty. The resulting statistic follows a t**-distribution** rather than the standard normal.

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$\implies$ With small samples, we need wider intervals to compensate for estimating σ.

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$\Longrightarrow$ In small samples, this interval would fail to capture β_2 more than 5% of the time. The t-distribution corrects for this.

Step 4: Rearranging for the Interval

Start with the probability statement:

$$P\left(-t_c \;\le\; \frac{b_2 - \beta_2}{\mathrm{se}(b_2)} \;\le\; t_c\right) = 1 - \alpha$$

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This is the $100(1-\alpha)\%$ **confidence interval** for β_2.

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Three moving parts, and you already know two of them (b_k and $\text{se}(b_k)$). The only new ingredient is t_c, which you look up in a table or compute in software.

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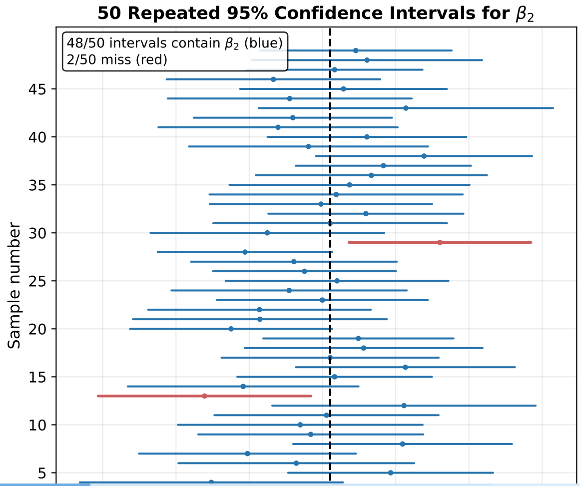
$\implies$ Our confidence is in the **procedure**, not in any single interval.

Visualizing Repeated Sampling

Simulation: draw 50 random samples from the food expenditure DGP, compute a 95% CI for β_2 from each.



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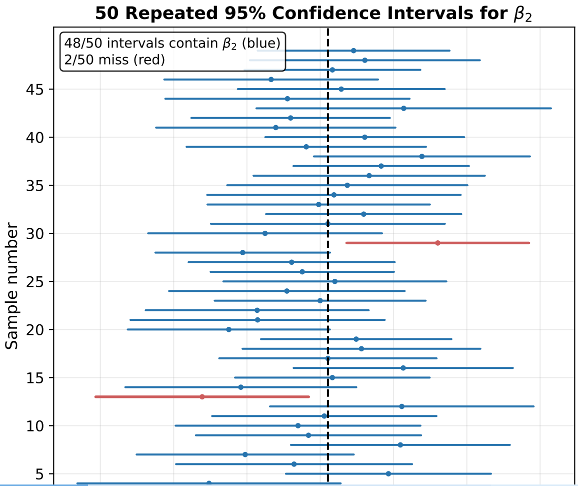

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Analogy: a factory produces light bulbs, and 95% work. You buy one. You do not say "there is a 95% probability this bulb works." It either works or it does not. The 95% describes the **factory's track record**, not your particular bulb.

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We estimate with 95% confidence that an additional $100 of weekly income increases food expenditure by between $5.98 and $14.44.

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95% ($\alpha = 0.05$)	$t_{(0.975,38)} = 2.024$	$[5.98,\ 14.44]$	8.46
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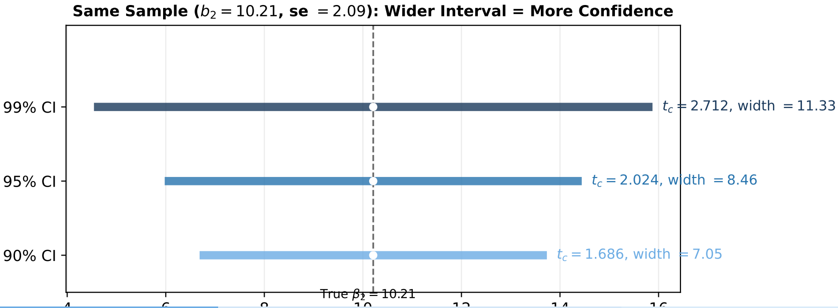

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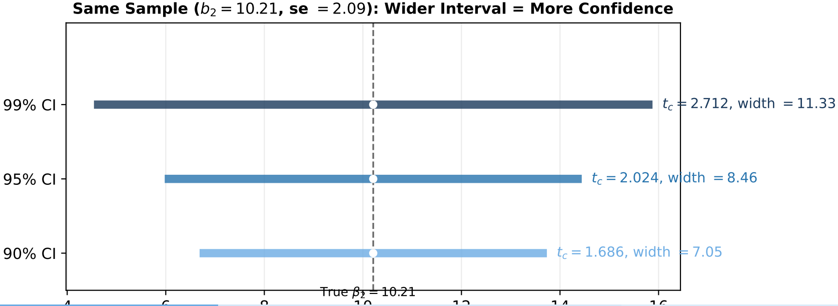

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We predict $287.61 per week. But how precise is this?

Variance of a Linear Combination

For $\hat{\lambda} = c_1 b_1 + c_2 b_2$, the variance of a sum of correlated random variables is:

$$\mathrm{Var}(\hat{\lambda}) = c_1^2 \ \mathrm{Var}(b_1) + c_2^2 \ \mathrm{Var}(b_2) + 2\, c_1 c_2 \ \mathrm{Cov}(b_1, b_2)$$

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For $c_1 = 1$, $c_2 = 20$:

$$\begin{aligned} \widehat{\text{Var}}(\hat{\lambda}) &= (1)^2 \times 1884.44 + (20)^2 \times 4.38 + 2(1)(20)(-85.90) \\ &= 1884.44 + 1752.72 - 3436.13 \\ &= 201.03 \end{aligned}$$

Variance of a Linear Combination

For $\hat{\lambda} = c_1 b_1 + c_2 b_2$, the variance of a sum of correlated random variables is:

$$\text{Var}(\hat{\lambda}) = c_1^2 \ \text{Var}(b_1) + c_2^2 \ \text{Var}(b_2) + 2\, c_1 c_2 \ \text{Cov}(b_1, b_2)$$

Do not forget the **covariance term**. The estimators b_1 and b_2 are correlated (both come from the same data), so you cannot just add the individual variances.

Where do the numbers come from? Software reports the estimated covariance matrix:

$$\widehat{\text{Cov}}(b_1, b_2) = \begin{pmatrix} \widehat{\text{Var}}(b_1) & \widehat{\text{Cov}}(b_1, b_2) \\ \widehat{\text{Cov}}(b_1, b_2) & \widehat{\text{Var}}(b_2) \end{pmatrix} = \begin{pmatrix} 1884.44 & -85.90 \\ -85.90 & 4.38 \end{pmatrix}$$

For $c_1 = 1$, $c_2 = 20$:

$$\begin{aligned} \widehat{\text{Var}}(\hat{\lambda}) &= (1)^2 \times 1884.44 + (20)^2 \times 4.38 + 2(1)(20)(-85.90) \\ &= 1884.44 + 1752.72 - 3436.13 \\ &= 201.03 \end{aligned}$$

$\text{se}(\hat{\lambda}) = \sqrt{201.03} = 14.18$

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This interval reflects our uncertainty about **both** β_1 and β_2 simultaneously, plus their covariance.

Outline

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Next: now that we can build intervals, what if we want a yes-or-no answer? $\implies$ hypothesis testing (Topic 10).

Thank you!

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