Hypothesis Testing

Is the Relationship Real, or Just Noise?

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Outline

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Where We Left Off

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But what if someone asks a sharper question:

"Does income *really* affect food spending, or could $\beta_2 = 0$?"

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Two regressions estimated from different datasets:

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Coefficient (b_2)	12.50	2.10
Std. Error ($\text{se}(b_2)$)	8.40	0.35

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$\Longrightarrow$ We need a formal measure of **evidence relative to noise**. That is what a hypothesis test provides.

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Two questions we will answer today:

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2. Does income raise food spending by more than $5.50 per $100? (Is $\beta_2 > 5.5$?)

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$\Longrightarrow$ Same data, same estimates, but different questions yield different answers.

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2. **Alternative hypothesis** (H_1): what we accept if we reject H_0
3. **Test statistic**: a number summarizing the evidence against H_0
4. **Rejection region** (or p-value): a decision rule
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Think of it as a trial: H_0 is "innocent until proven guilty." We need strong evidence to convict.

The Null and Alternative Hypotheses

Null hypothesis: $H_0 : \beta_k = c$, where c is a specific number.

We write β_k for any coefficient in the model; in our example, $k = 2$.

The null always contains an equality. It is our "default" belief until evidence says otherwise.

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Alternative	**When to use**
$H_1 : \beta_k > c$	Theory predicts a positive effect
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Example: "Does income affect food spending?"

- $H_0 : \beta_2 = 0$ (income has no effect)
- $H_1 : \beta_2 \neq 0$ (income has *some* effect) $\leftarrow$ two-sided

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- The denominator $\mathrm{se}(b_k)$ **scales** this distance by the estimation precision
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$\Longrightarrow$ Large $|t|$ = evidence against H_0. But how large is "large enough"?

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Testing $H_0: \beta_2 = 7.5$ (Consultant's Claim)

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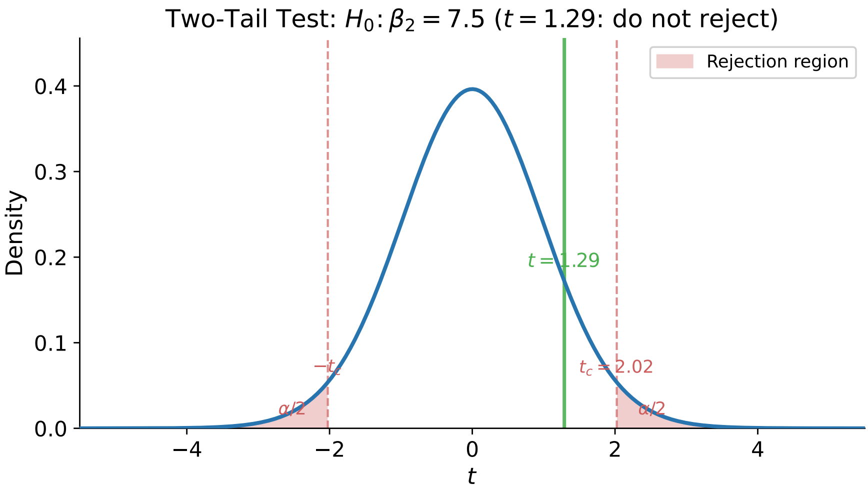
$\implies$ The data are consistent with $\beta_2 = 7.5$. But they are also consistent with $\beta_2 = 8.5$ ($t = 0.82$), or $\beta_2 = 6.5$ ($t = 1.77$). Not rejecting does **not** prove the null is true.

Visualizing a Non-Rejection



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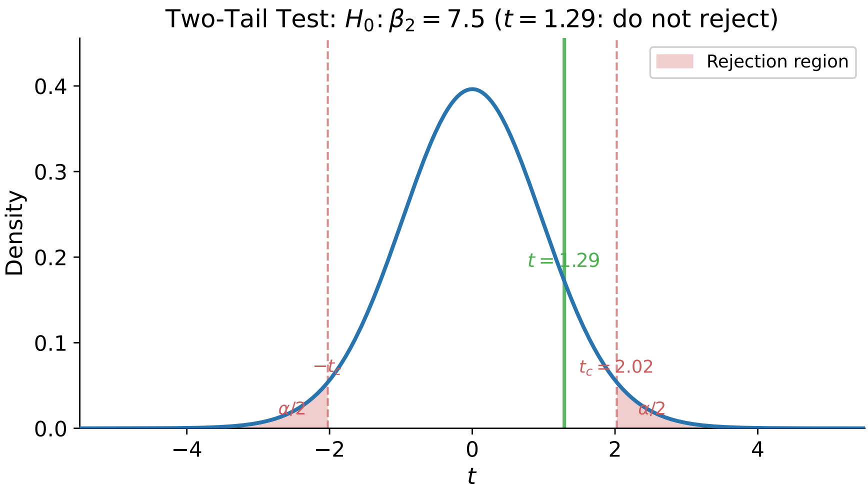

$t = 1.29$ falls in the non-rejection region. The estimate $b_2 = 10.21$ is close enough to 7.5, relative to

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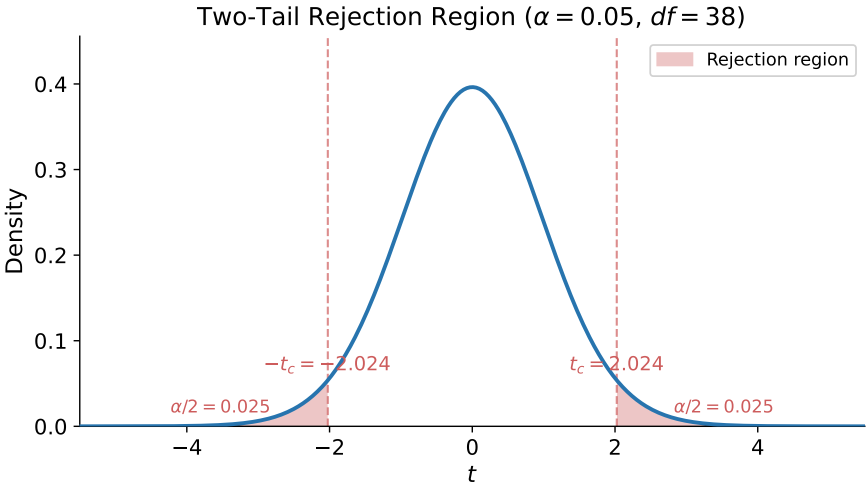
Yes. Reject H_0. There is a statistically significant relationship between income and food expenditure.

Visualizing the Rejection Region



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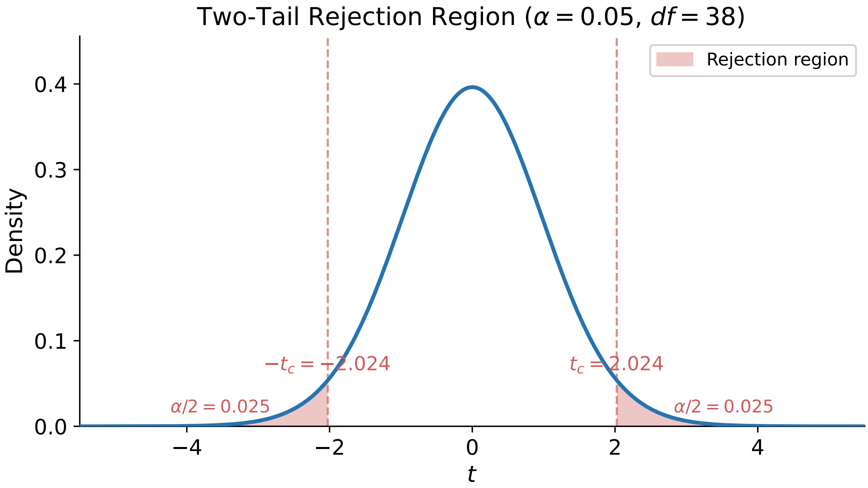

With $t = 4.88$, we land deep in the rejection region. The data are not compatible with $\beta_2 = 0$.

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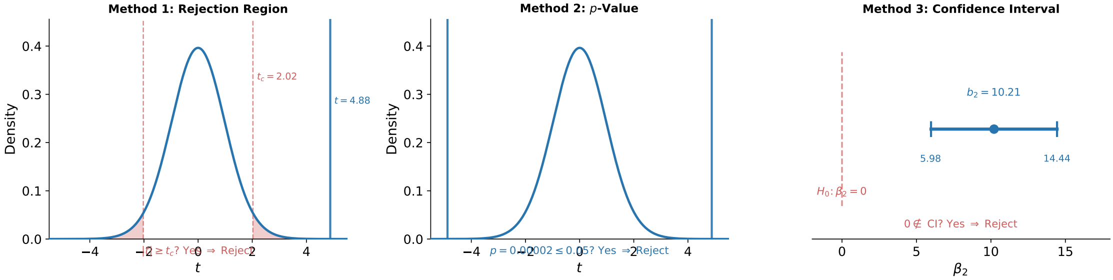
$\implies$ These are not three different tests. They are three windows onto the **same test**.

Three Methods: Visual Comparison



All three panels represent the same test ($H_0 : \beta_2 = 0$, $\alpha = 0.05$). Each reaches “Reject” through a different lens: comparing t to t_c, comparing p to α, or checking whether 0 falls inside the CI.

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And the p-value $\leq \alpha$ precisely when $|t| \geq t_c$.

$\implies$ For a two-sided test at level α: reject via $|t|$, reject via p-value, reject via CI are all the same statement, rearranged.

Comparing the Two Tests

	Test 1: Consultant	**Test 2: Significance**
H_0	$\beta_2 = 7.5$	$\beta_2 = 0$
H_1	$\beta_2 \neq 7.5$	$\beta_2 \neq 0$
t	$\frac{10.21 - 7.5}{2.09} = 1.29$	$\frac{10.21 - 0}{2.09} = 4.88$
p-value	0.2033	0.00002
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$\Longrightarrow$ The same b_2 and $\text{se}(b_2)$ produce opposite conclusions because the **distance to the null** differs. The t-statistic measures how many standard errors b_2 is from c.

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Decision rule:

$$p \leq \alpha \implies \text{Reject } H_0 \qquad p > \alpha \implies \text{Do not reject } H_0$$

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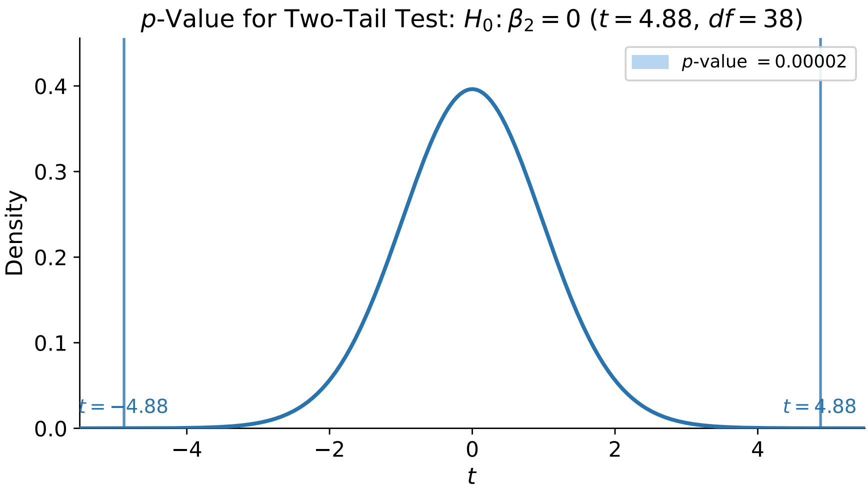
Software regression output reports the two-tail p-value for $H_0 : \beta_k = 0$ by default. If you need a one-tail p-value: divide by 2, **but only when the sign of t agrees with your alternative** (e.g., $t > 0$ for $H_1 : \beta_k > 0$). If t points in the opposite direction, the one-tail $p > 0.50$ and you cannot reject.

p-Value for $H_0 : \beta_2 = 0$ (Two-Tail)



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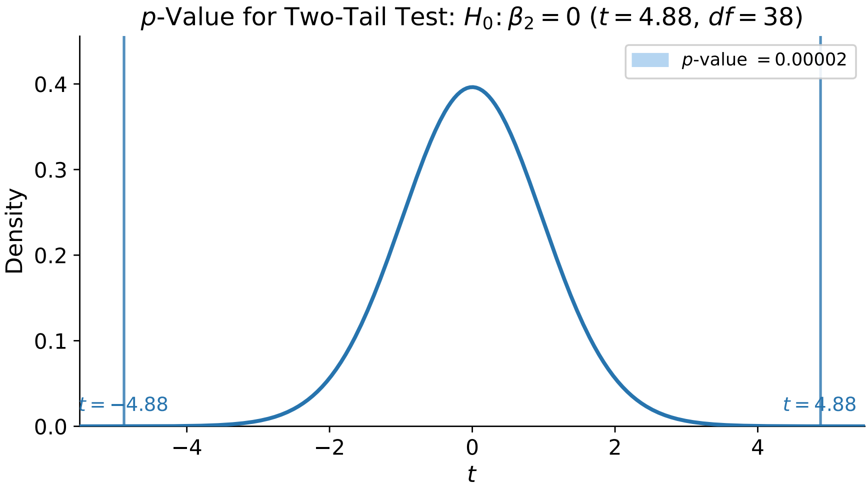

$p = 0.00002$. The shaded area is so small it is barely visible. At any conventional α (0.01, 0.05, 0.10),

Outline

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Memory rule: The rejection region is in the direction of the inequality in H_1.

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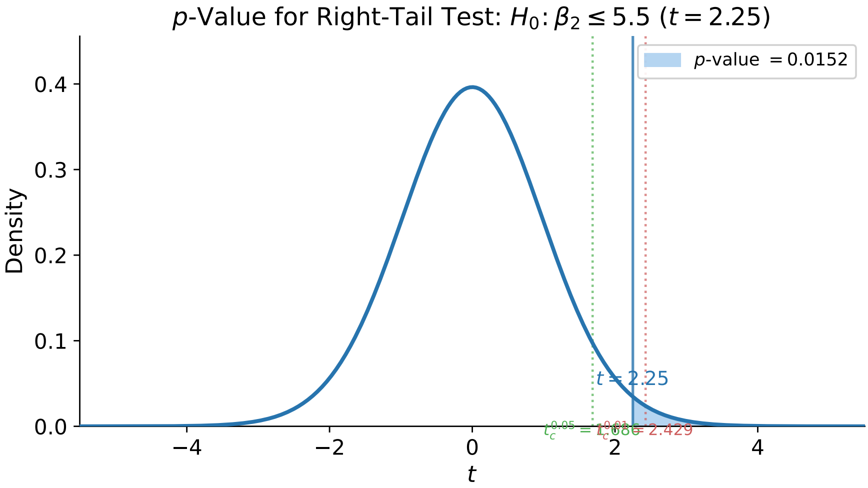
Why $\alpha = 0.01$? The cost of a Type I error (building an unprofitable store) is high. A conservative α raises the evidence bar.

p-Value for the Right-Tail Test



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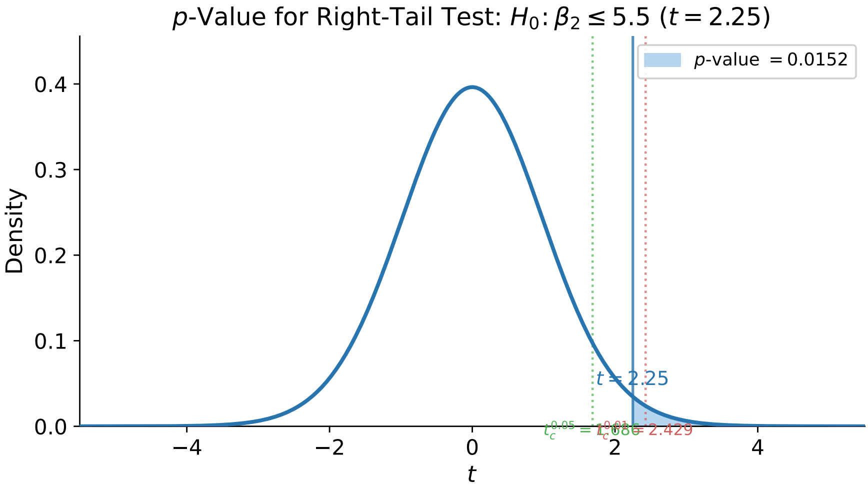

$p = 0.0152$. At $\alpha = 0.01$: $0.0152 > 0.01$, do not reject. At $\alpha = 0.05$: $0.0152 < 0.05$, reject. The

One-Tail vs. Two-Tail: Comparison

Testing $H_0 : \beta_2 = 0$ with $\alpha = 0.05$:

	Two-Tail	**Right-Tail**
H_1	$\beta_2 \neq 0$	$\beta_2 > 0$
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$\implies$ Use one-sided only when theory gives a clear directional prediction *before* seeing the data. Otherwise, use two-sided.

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	H_0 **is actually true**	H_0 **is actually false**
Reject H_0	Type I error	Correct
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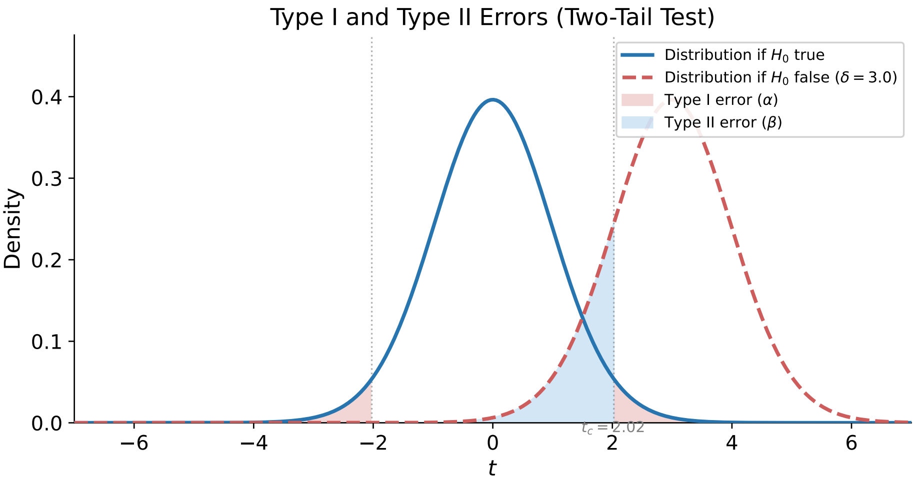
Power $= 1 - \beta =$ probability of correctly rejecting a false H_0.

Type I and Type II: Visual



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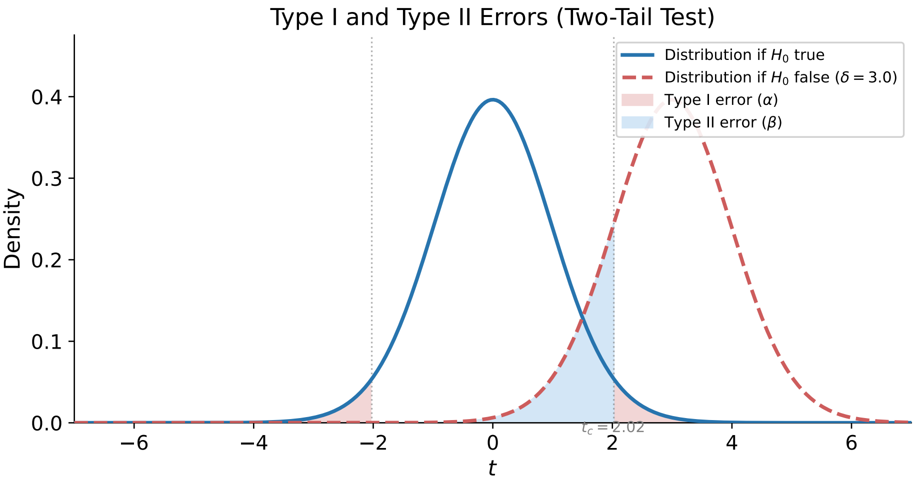

Lowering α (moving t_c outward) shrinks the Type I region but **enlarges** the Type II region. There is always a tradeoff.

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$\Longrightarrow$ “Do not reject” is weaker than “reject.” It means the data cannot distinguish β_2 from zero, not that β_2 *is* zero.

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If we lower α...	Type I	Type II
$\alpha = 0.10$	10%	Lower
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$\implies$ The choice of α should reflect the relative costs of the two types of errors in your specific context.

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$\Longrightarrow$ Failing to reject does not single out $\beta_2 = 7.5$ as the truth. It just says 7.5 is **one of many values** the data cannot distinguish from $b_2 = 10.21$.

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$\implies$ Always report the **magnitude** of b_k alongside the t-statistic. Significance tells you whether the effect is distinguishable from zero; it does not tell you whether the effect is large enough to care about.

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Software Output: What to Look For

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$\Longrightarrow$ INCOME: $p \approx 0.0000$ means the relationship is significant at any conventional level.
$\Longrightarrow$ Intercept: $p = 0.0622$ means not significant at 5%, but significant at 10%.

Thank you!

jakeanderson@g.ucla.edu