Prediction Intervals and Goodness of Fit

How Much Can Our Predictions Be Off, and How Well Does the Model Fit?

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$\Longrightarrow$ Today: prediction intervals, R^2, and what they tell us (and what they don't).

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1. Prediction Intervals
2. Decomposing Total Variation: SST = SSR + SSE
3. R^2: The Coefficient of Determination
4. Effects of Data Scaling
5. Summary

Two Questions About Prediction

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$\Longrightarrow$ Question 2 is harder. The answer is always **less precise**.

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Under our standard regression assumptions, $\hat{y}_0$ is the **Best Linear Unbiased Predictor** (BLUP): no other linear unbiased predictor has a smaller forecast error variance.

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$\Longrightarrow$ The prediction variance has an **extra** σ^2 from the individual error e_0. This term does not shrink

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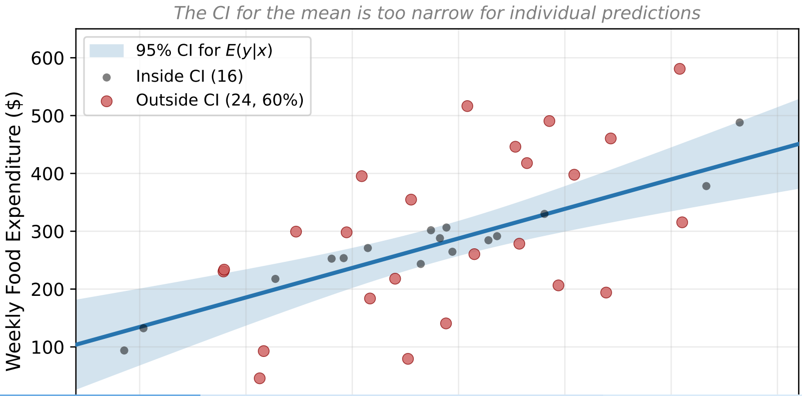

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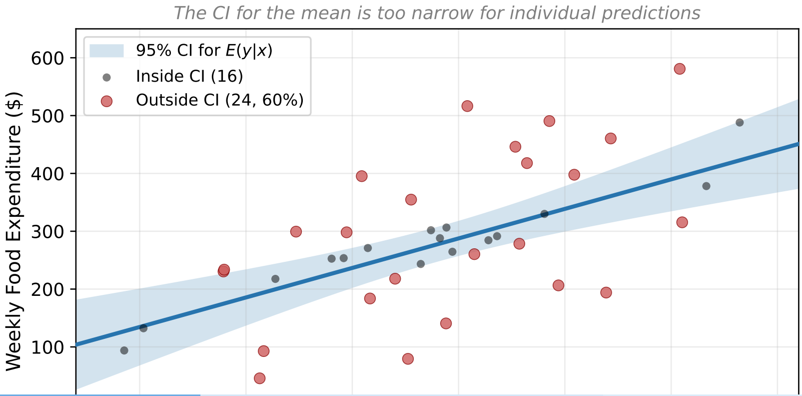

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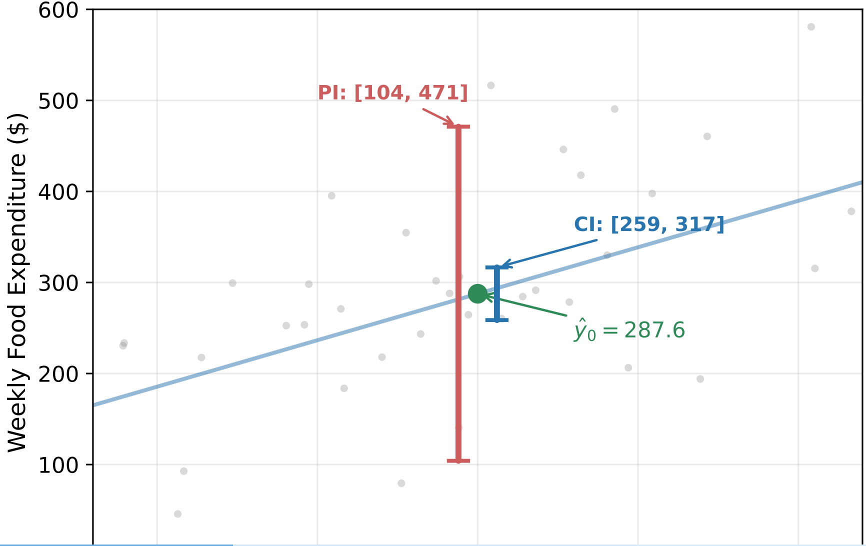
$\Longrightarrow$ We predict between \$104 and \$471. That range is enormous.

Why Prediction Intervals Are Wider Than CIs for the Mean



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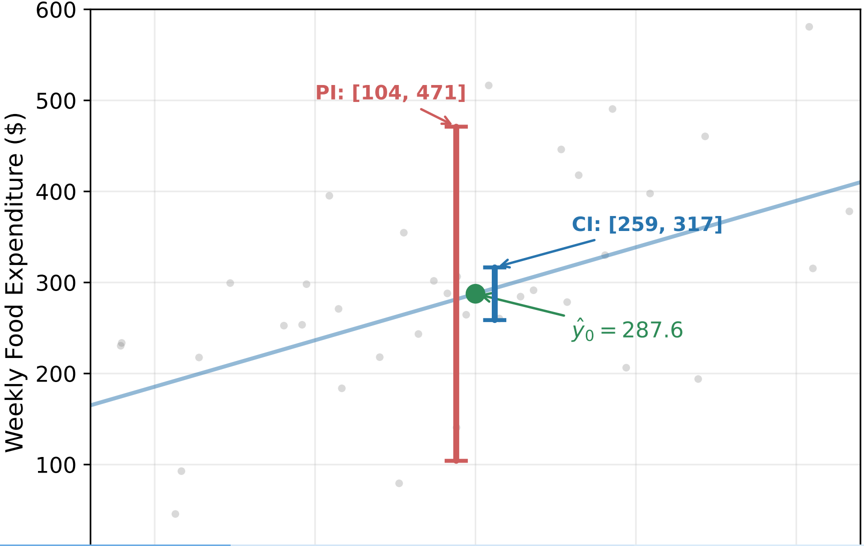
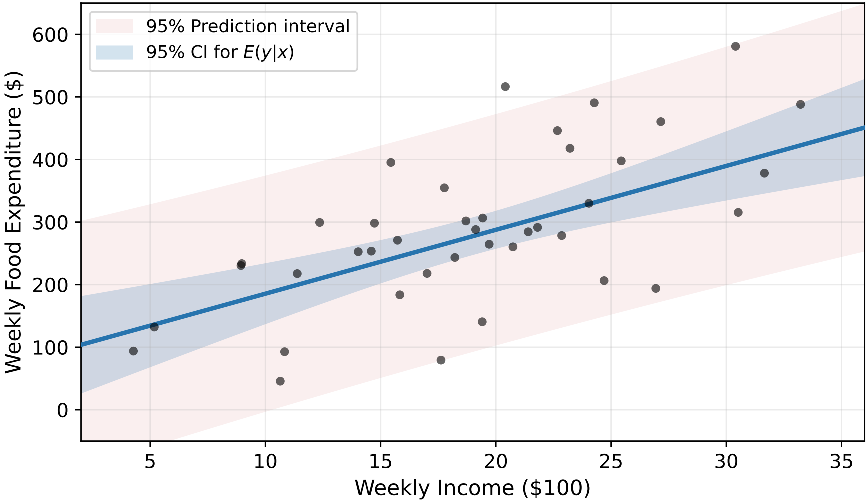


Both Intervals on the Same Plot



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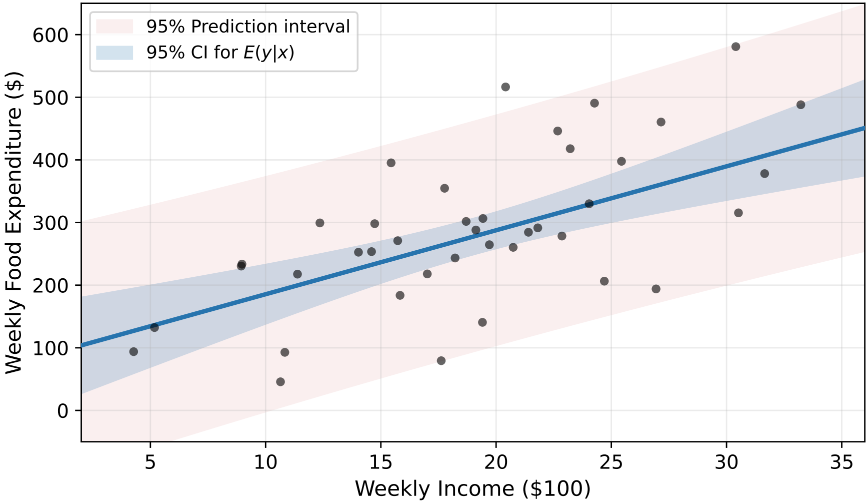

The blue band (CI for the mean) shrinks toward the line as $N \to \infty$. The red band (prediction

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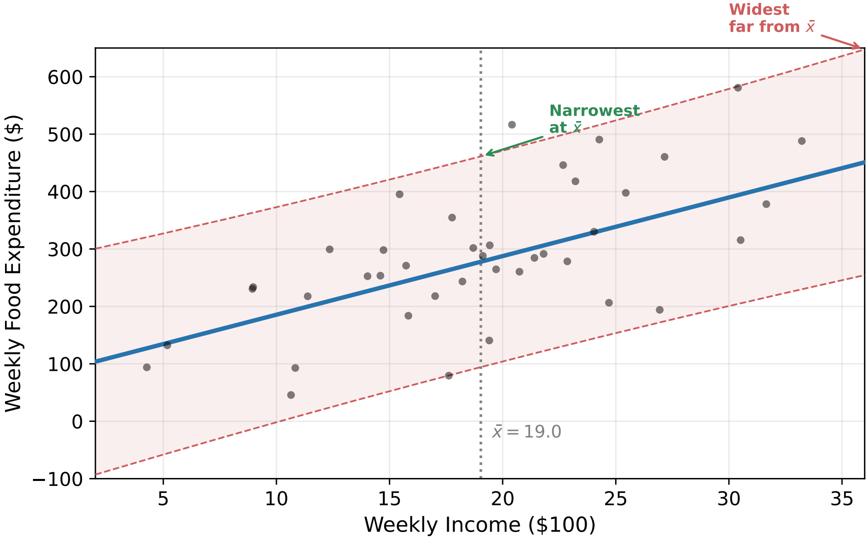

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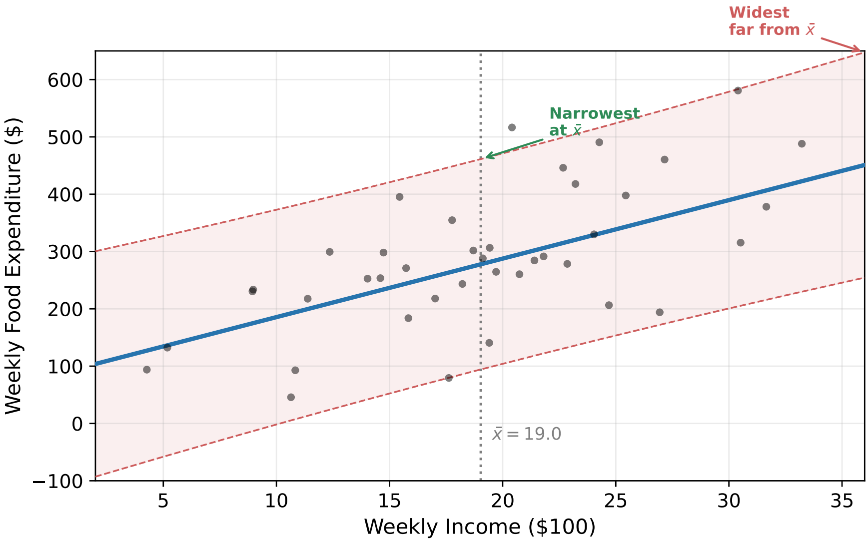

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$\implies$ Prediction intervals never collapse to a point. They converge to $\hat{y}_0 \pm t_c \cdot \sigma$ as $N \to \infty$.

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To answer this, we decompose the total variation in y into two pieces:

- The part explained by the regression
- The part left unexplained (residuals)

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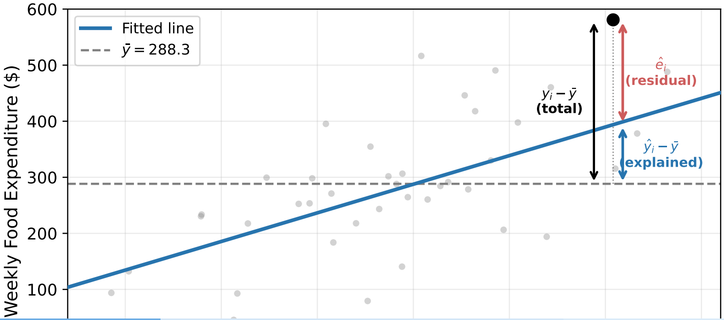

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Square and sum over all observations:

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This identity requires an **intercept** in the model. The cross-product $\sum(\hat{y}_i - \bar{y})\hat{e}_i$ vanishes because the OLS normal equations guarantee $\sum \hat{e}_i = 0$ and $\sum x_i\hat{e}_i = 0$.

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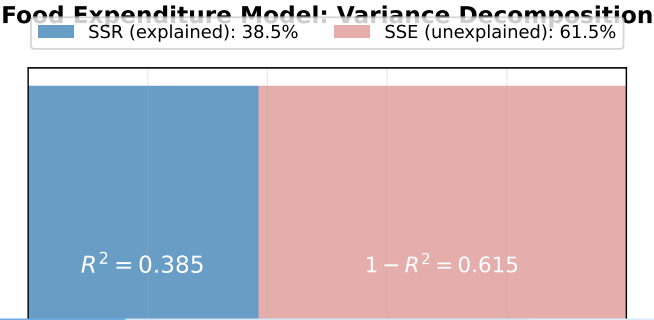

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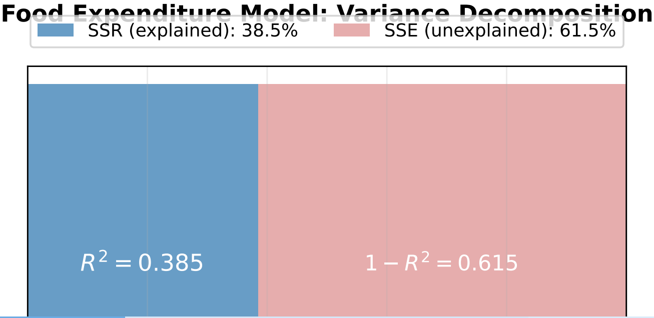

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For the food expenditure model:

$$R^2 = 1 - \frac{304{,}505}{495{,}132} = 0.385$$

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R^2 can also be expressed as:

$$R^2 = \text{Corr}(y_i, \hat{y}_i)^2$$

the squared correlation between observed and fitted values. This version generalizes to multiple regression.

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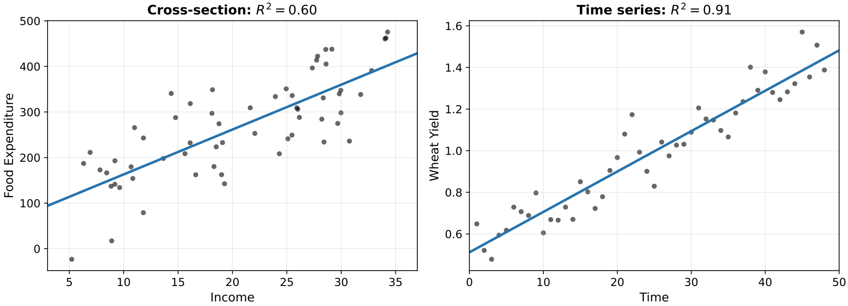

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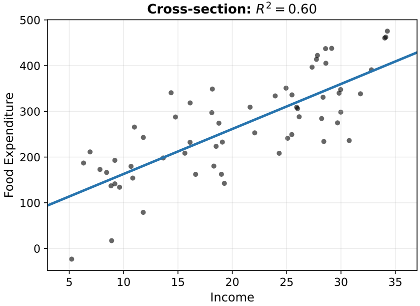

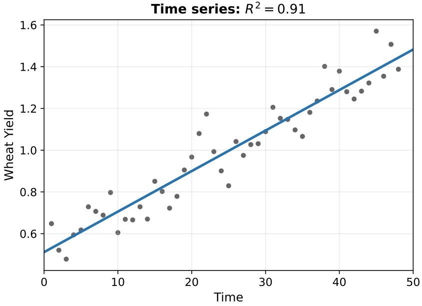

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2. **The coefficients are meaningful.** An insignificant slope ($t < 2$) in a model with $R^2 = 0.90$ from a strong time trend means x is not doing the explaining.

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$\Longrightarrow$ Evaluate a model by its coefficients (signs, magnitudes, significance), not by R^2 alone.

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$\Longrightarrow$ Scaling is a change in **units**, not a change in the relationship. Statistical conclusions (t-stats, p-values, R^2) are invariant to units.

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$\Longrightarrow$ Prediction intervals, R^2, coefficient significance, and residual diagnostics are complementary. No single number summarizes model quality.

Thank you!

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