Hypothesis Testing in Multiple Regression

Same Logic, More Coefficients, New Questions

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Outline

1. From SLR to MR: What Changes?
2. Individual t-Tests in MR
3. Marginal Effects with Polynomial Terms
4. Testing Linear Combinations of Coefficients
5. The General Linear Combination Formula
6. Individual $\neq$ Joint Significance
7. Summary

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$\Longrightarrow$ If you can do t-tests in SLR, you can do them in MR. Today is about the new questions MR opens up.

The Wage Model

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That seems unlikely. We probably need **diminishing returns to experience**.

$\Longrightarrow$ Add EXPER^2 to allow the marginal effect of experience to change:

$$\text{WAGE}_i = \beta_1 + \beta_2\,\text{EDUC}_i + \beta_3\,\text{EXPER}_i + \beta_4\,\text{EXPER}_i^2 + e_i$$

The Estimated Model

Using the CPS data ($N = 1000$, $K = 4$, $df = 996$):

$$\widehat{\text{WAGE}} = \underset{(2.87)}{-10.40} + \underset{(0.16)}{2.44}\ \text{EDUC} + \underset{(0.11)}{0.55}\ \text{EXPER} \underset{(0.0020)}{-0.0063}\ \text{EXPER}^2$$

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Today's questions:

1. Are individual coefficients statistically significant?
2. What is the **marginal return to experience** at a specific level, and is it significant?
3. Does "individually significant" mean "jointly significant"?

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$$t = \frac{b_k}{\text{se}(b_k)}$$

Rejection rules are the same as before:

- Two-tail: reject if $|t| \geq t_{(1-\alpha/2,\ N-K)}$
- Right-tail ($H_1 : \beta_k > c$): reject if $t \geq t_{(1-\alpha,\ N-K)}$
- Left-tail ($H_1 : \beta_k < c$): reject if $t \leq -t_{(1-\alpha,\ N-K)}$

Example: Is Education Significant?

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$\implies$ This is no different from what you did in SLR. The only change is the degrees of freedom (996 instead of 998).

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But notice: $b_4 = -0.0063$ looks tiny. Is it economically meaningful? We need to think about what the **marginal effect** of experience actually is.

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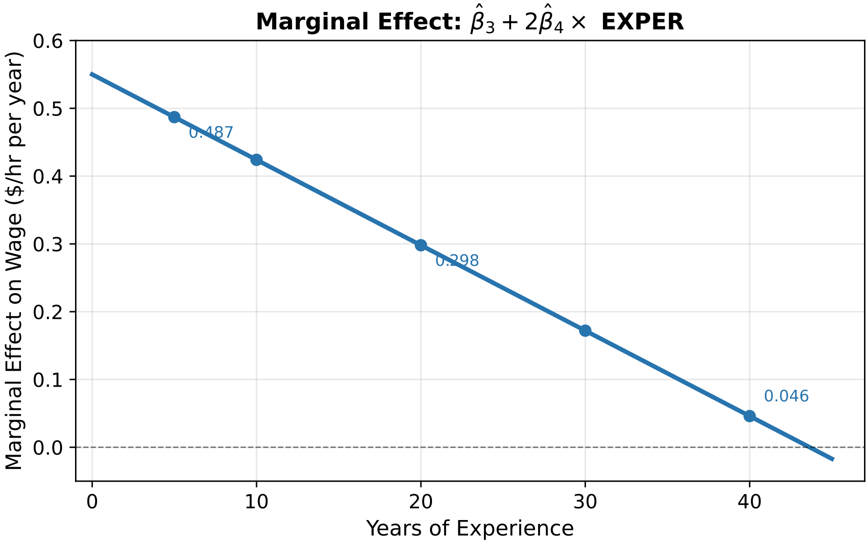
Experience level	**Marginal effect ($/hr)**
5 years	$0.55 - 0.063 = 0.487$
10 years	$0.55 - 0.126 = 0.424$
20 years	$0.55 - 0.252 = 0.298$
30 years	$0.55 - 0.378 = 0.172$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Marginal Effect on Wage ($/hr per year)
Years of Experience
0.487
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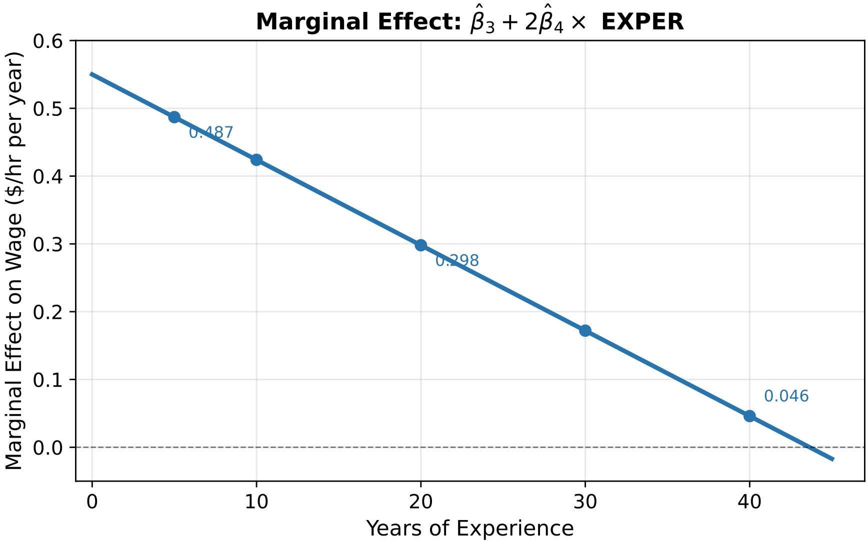
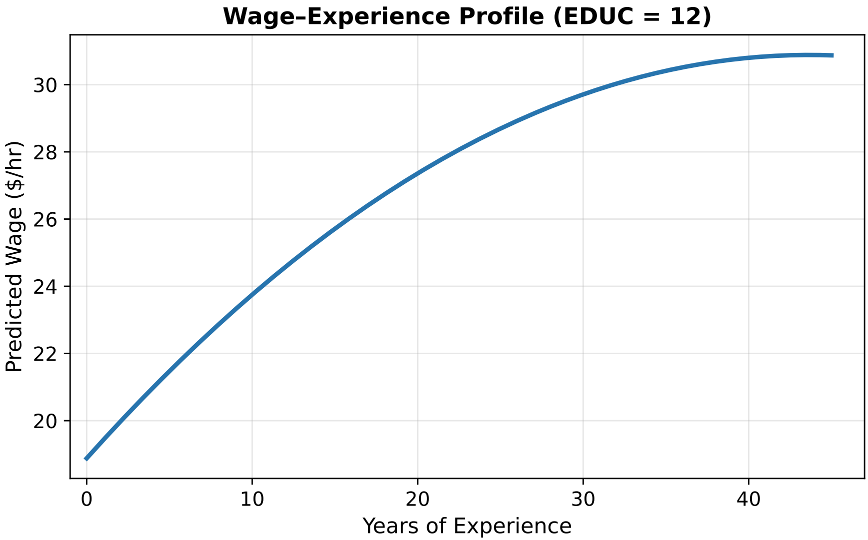
0.046
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
10
20
30
40

Visualizing Diminishing Returns



Visualizing the Wage–Experience Profile



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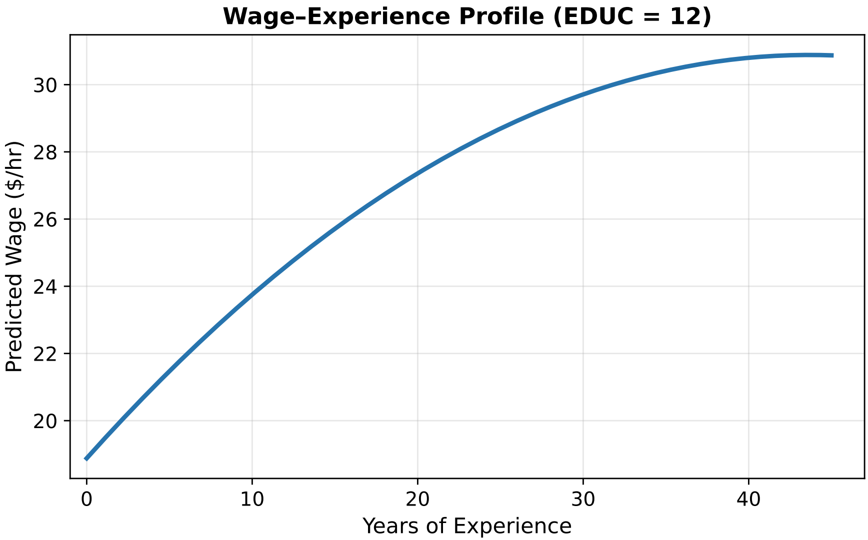

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$\implies$ To test the marginal effect at a specific experience level, we need a test for a **linear combination** of coefficients.

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$\Longrightarrow$ We can test $H_0 : \theta = 0$ using the same t-test framework. We just need the standard error of $\hat{\theta}$.

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Where does $\widehat{\text{Cov}}(b_3, b_4)$ come from?

$\implies$ The **estimated covariance matrix** of the coefficients. Software computes this for every regression. You just need to read off the right entry.

The Covariance Matrix

Software reports a matrix with variances on the diagonal and covariances off the diagonal:

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$$\widehat{\text{Cov}}(b_1, b_2, b_3, b_4) = \begin{bmatrix} 8.2369 & -0.3984 & -0.0639 & 0.00040 \\ -0.3984 & 0.0256 & 0.0001 & -0.00001 \\ -0.0639 & 0.0001 & 0.0121 & -0.00022 \\ 0.00040 & -0.00001 & -0.00022 & 0.0000040 \end{bmatrix}$$

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Reading the matrix:

- $\widehat{\text{Var}}(b_3) = 0.0121 \implies \text{se}(b_3) = \sqrt{0.0121} = 0.11$
- $\widehat{\text{Var}}(b_4) = 0.0000040 \implies \text{se}(b_4) = 0.0020$
- $\widehat{\text{Cov}}(b_3, b_4) = -0.00022$

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Step 2: Standard error.

$$\begin{aligned}\widehat{\text{Var}}(\hat{\theta}) &= (1)^2(0.0121) + (20)^2(0.0000040) + 2(1)(20)(-0.00022) \\ &= 0.0121 + 0.0016 - 0.0088 \\ &= 0.0049\end{aligned}$$

$$\text{se}(\hat{\theta}) = \sqrt{0.0049} = 0.070$$

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Step 3: t-statistic.

$$t = \frac{0.424 - 0}{0.070} = 6.06$$

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$$\text{se}(\hat{\theta}) = \sqrt{0.0049} = 0.070$$

Step 3: t-statistic.

$$t = \frac{0.424 - 0}{0.070} = 6.06$$

$|6.06| \geq 1.962?$ Yes. $p \approx 0.000.$

Testing the Marginal Effect at 10 Years

$H_0: \beta_3 + 20\beta_4 = 0 \quad$ vs. $\quad H_1: \beta_3 + 20\beta_4 \neq 0 \qquad (\alpha = 0.05)$

Step 1: Point estimate.

$$\hat{\theta} = b_3 + 20b_4 = 0.55 + 20(-0.0063) = 0.424$$

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Reject H_0. At 10 years of experience, the marginal return is significantly different from zero.

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Still highly significant ($p \approx 0.000$). Even at 30 years, each additional year of experience still raises wages.

Comparison: How the Test Changes with Experience

EXPER	$\hat{\theta}$	se($\hat{\theta}$)	t	**Significant?**
5	0.487	0.090	5.41	Yes
10	0.424	0.070	6.06	Yes
20	0.298	0.030	9.93	Yes
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$\implies$ The marginal return to experience shrinks as experience grows. At 40 years, the estimated return is $0.05/hr and is no longer distinguishable from zero.

Notice the standard error is **not constant** across experience levels. It depends on the covariance matrix and the weights c_3, c_4. The standard error is smallest near the center of the data and rises at extreme values.

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$\implies$ This is the same t-test structure. The only new ingredient is computing $\text{se}(\hat{\lambda})$ from the covariance matrix.

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How much more does a worker with 16 years of education earn than one with 12 years, holding experience constant?

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$\Longrightarrow$ Four extra years of college are associated with \$8.50 to \$11.02 more per hour. No covariance terms needed here because only one coefficient is involved.

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$\Longrightarrow$ Each variable's t-test asks: "Does this variable contribute, *given that the other is already in the model*?"

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$$H_0 : \beta_3 = 0 \text{ AND } \beta_4 = 0$$

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This is a **joint hypothesis**, and the t-test cannot handle it.

$\Longrightarrow$ We need the ***F*-test**, which is the topic for next lecture. For now, remember: individually insignificant does **not** mean jointly insignificant.

Preview: Why the F-Test Exists

Question	**Test**	**Statistic**
Is one coefficient zero?	t-test	$t = \frac{b_k}{\text{se}(b_k)}$
Are *several* coefficients zero?	F-test	Next lecture

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$\Longrightarrow$ The t-test handles one constraint. The F-test handles many.

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What Changed from SLR to MR

	SLR	**MR**
t-statistic formula	$\dfrac{b_k - c}{\text{se}(b_k)}$	Same
Degrees of freedom	$N - 2$	$N - K$
Test individual coefficients?	Yes	Yes
Test linear combinations?	Not needed	Yes (uses cov matrix)
Joint tests?	N/A	Need F-test

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Polynomial terms (like EXPER^2):

- The marginal effect is a **derivative**, not a single coefficient
- Testing the marginal effect at a specific point requires a linear combination test
- Individual t-tests on correlated terms can both be insignificant even when the variables are jointly significant

Thank you!

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