Interaction Terms

When One Variable's Effect Depends on Another

Jake Anderson

March 21, 2026

Outline

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A Question About Wages

Does the return to education differ for men and women?

Two possibilities:

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These require different models. Our current tools (Topics 1–15) can only handle the first case. Today we learn how to handle the second.

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The No-Interaction Model

Start with a model that allows wages to differ by gender, but forces the **same return to education**:

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{female}_i + e_i$$

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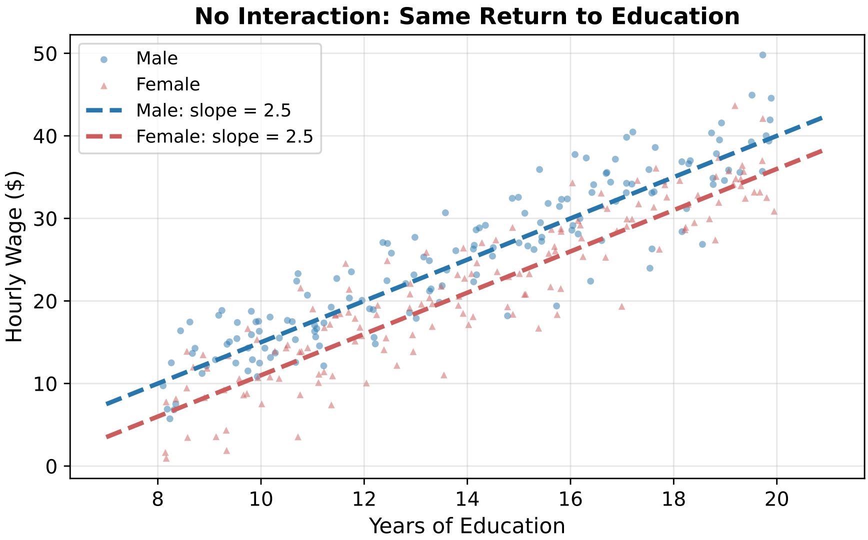
$\implies$ Two lines with the **same slope** (β_2) but different intercepts.

The dummy variable female shifts the line up or down, but the return to each extra year of education is identical for both groups.

Parallel Lines: The Picture



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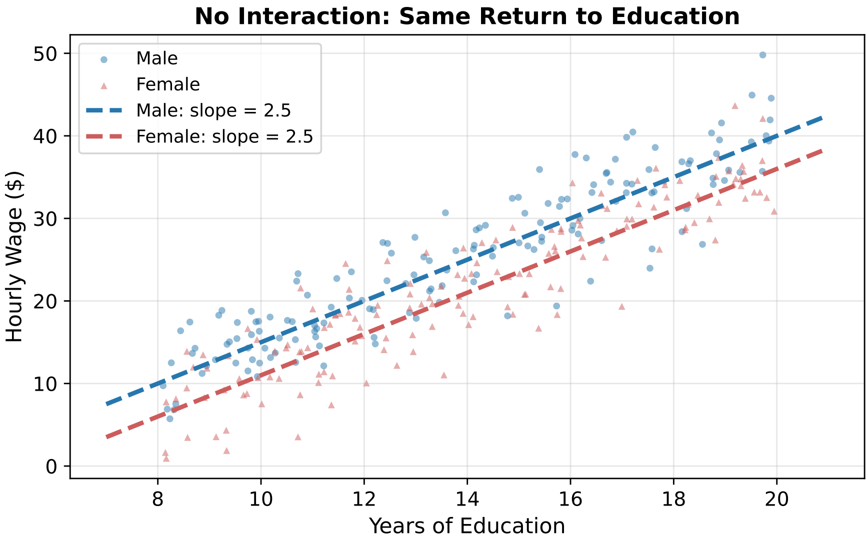

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But what if the labor market rewards men's education more steeply than women's?

Or what if women with graduate degrees close the gap more than women with only high school?

$\Longrightarrow$ We need a model that allows **different slopes**, not just different intercepts.

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Adding the Interaction Term

Create a new variable: female $\times$ educ.

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{female}_i + \beta_4\,(\text{female}_i \times \text{educ}_i) + e_i$$

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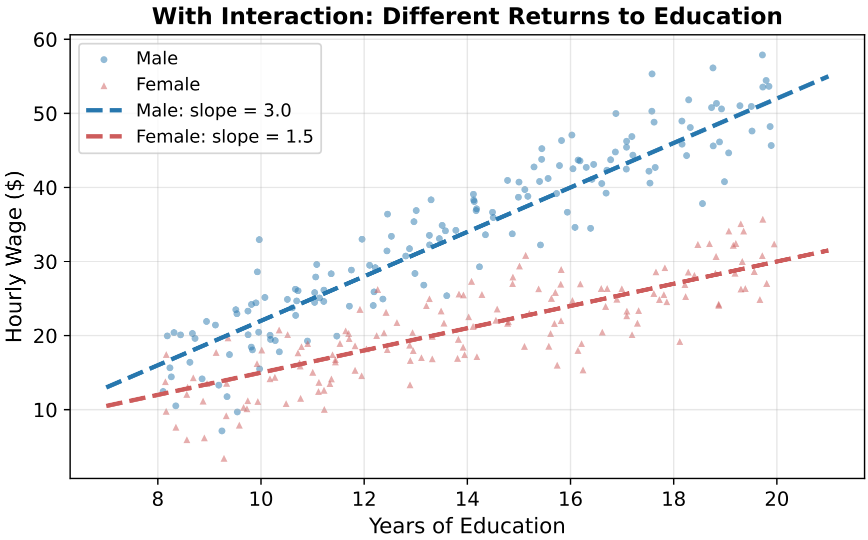
$\Longrightarrow$ Now each group has its **own intercept and its own slope.**

β_4 measures *how much the return to education differs* for women relative to men.

Different Slopes: The Picture



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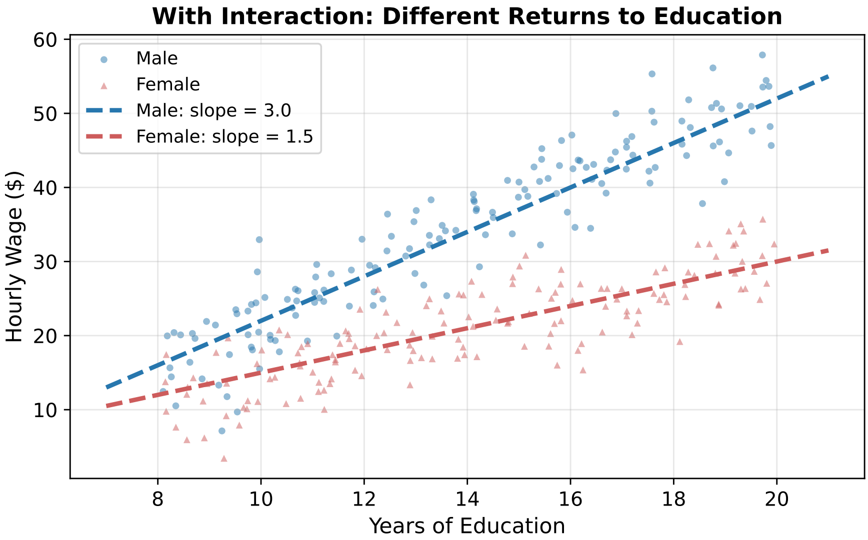
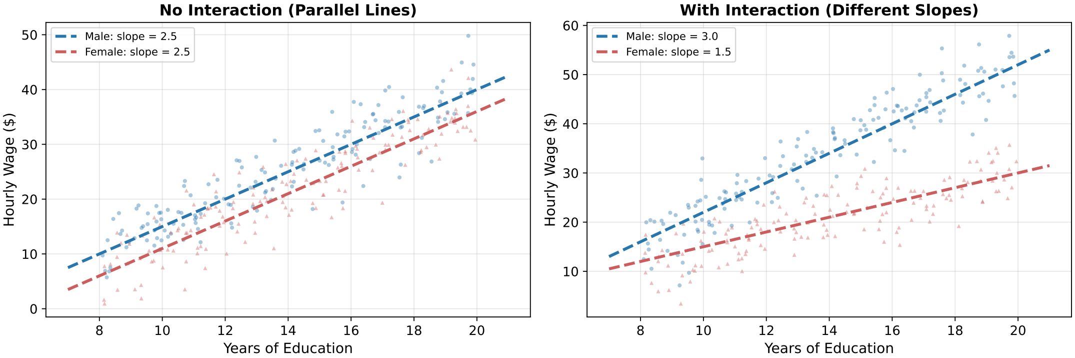


Side-by-Side Comparison



Interpreting the Interaction Coefficient

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What does each coefficient measure?

- β_2: return to education **for men** (the reference group)
- β_3: wage gap at $\text{educ} = 0$ (rarely meaningful on its own)
- β_4: **difference in returns** to education, women vs. men

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Interaction terms are not limited to dummy variables. The same logic applies when both variables are continuous (we will see this in detail shortly):

$$\widehat{\text{wage}} = -18.76 + 2.66\,\text{educ} + 0.24\,\text{exper} - 0.003\,(\text{educ} \times \text{exper})$$

Here $b_4 = -0.003$ tells us: a worker with one more year of education gets \$0.003/hr *less* return per year of experience.

Outline

The Marginal Effect Is No Longer a Single Number

Without interaction:

$$\text{wage} = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{female} + e \qquad \Longrightarrow \qquad \frac{\partial\ \text{E(wage)}}{\partial\,\text{educ}} = \beta_2$$

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$\Longrightarrow$ The marginal effect of education **depends on** whether the worker is female.

You cannot report a single number for “the effect of education” without specifying who you are talking about.

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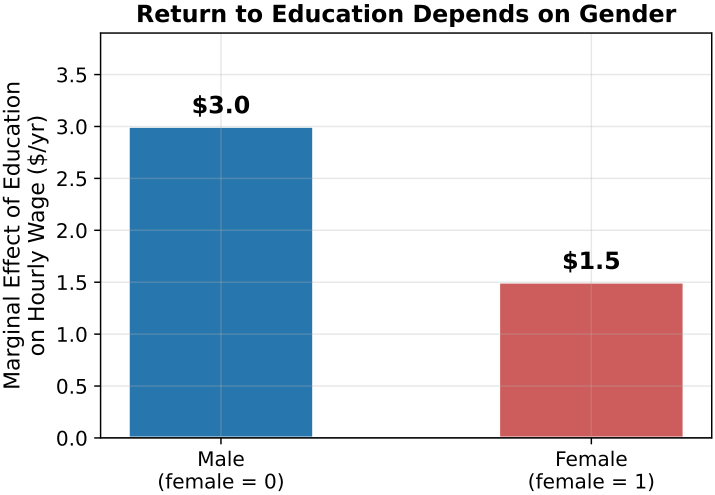

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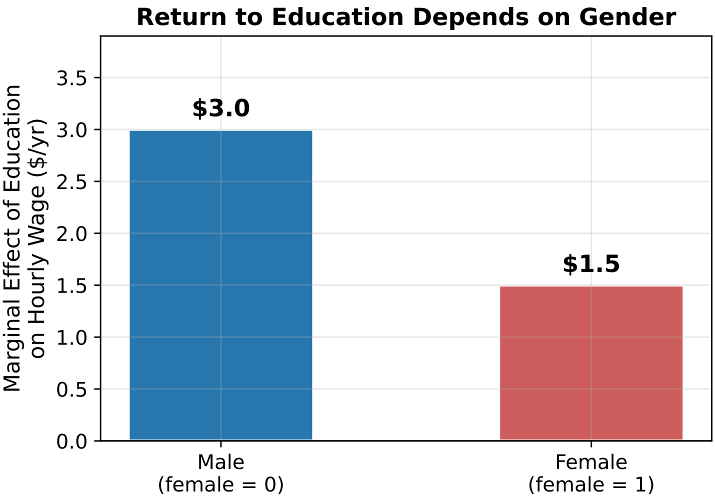

The return to education is β_2 for men and $\beta_2 + \beta_4$ for women. With $\beta_4 < 0$, women get a lower

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$\Longrightarrow$ Always ask: "The effect of x_2 *at what level of* x_3?"

Common choices: the sample mean, the 25th/75th percentile, or substantively meaningful values (e.g., 8 vs. 16 years of education).

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When Both Variables Are Continuous

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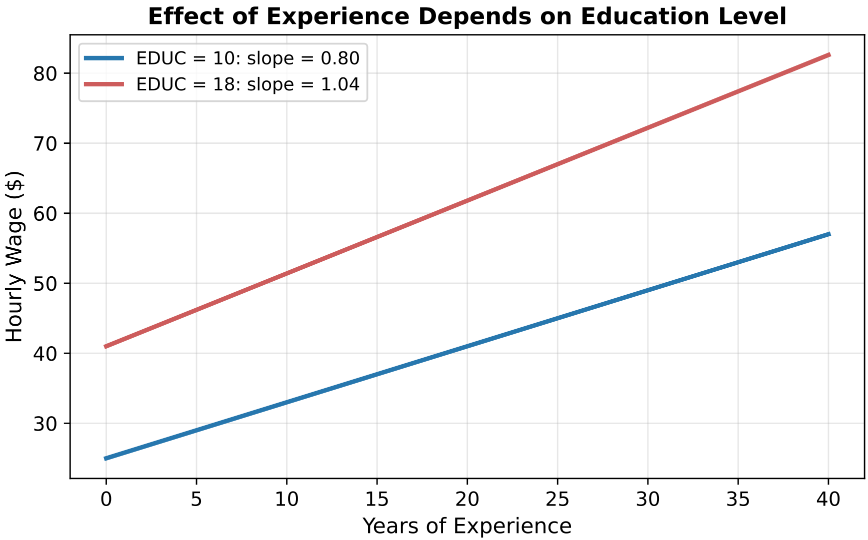
If $\beta_4 > 0$: education and experience are **complements**. More education amplifies the return to experience.

If $\beta_4 < 0$: they are **substitutes**. More education reduces the return to experience (as in the CPS data).

Visualizing the Continuous Interaction



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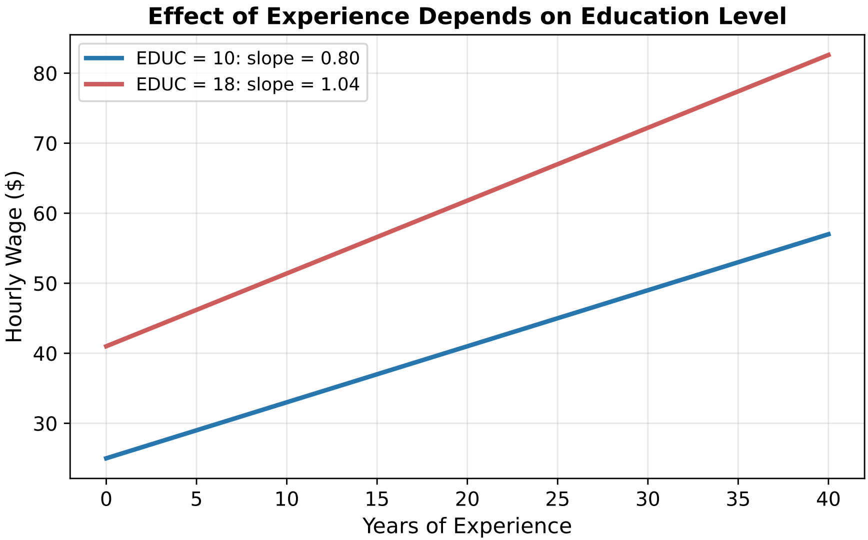
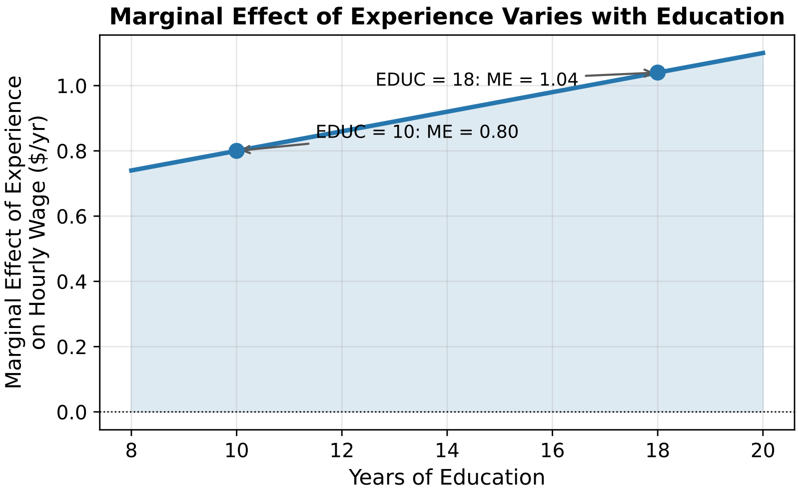


Marginal Effect of Experience Across Education Levels



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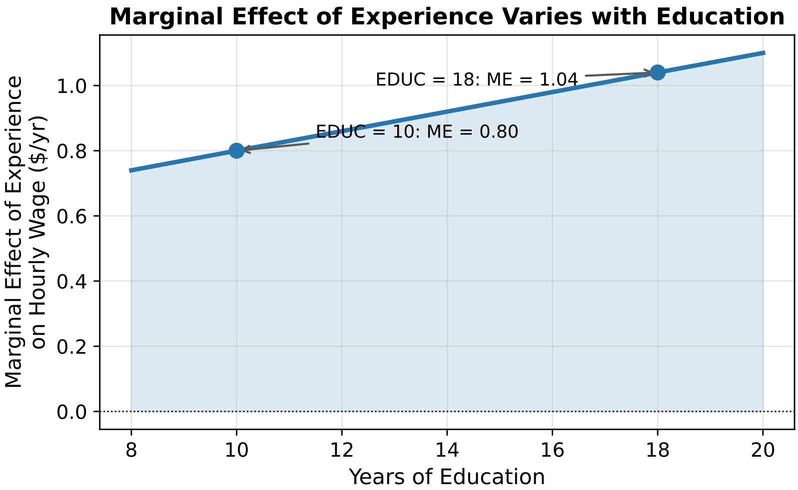

The marginal effect of experience is a **linear function** of education: $\beta_3 + \beta_4 \times \text{educ}$.

CPS Wage Equation (Textbook Example 5.15)

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Because $\hat{\beta}_4 < 0$, the wage–experience lines for different education levels **converge** rather than fan out.

The Log-Quadratic Wage Equation (Textbook Example 5.16)

A richer specification with $\ln(\text{wage})$ as the dependent variable:

$$\ln(\text{wage}) = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{exper} + \beta_4(\text{educ} \times \text{exper}) + \beta_5\,\text{exper}^2 + e$$

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Marginal effects (approximate % changes $\times 100$):

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The textbook (Table 5.4) reports that the return to education is 2.53 percentage points lower for someone with 20 years of experience vs. 0.

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For $y = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{exper} + \beta_4(\text{educ} \times \text{exper}) + e$:

1. Choose representative values of educ (e.g., 10, 14, 18)
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What to look for:

- **Parallel lines** $\implies$ no interaction ($\beta_4 \approx 0$)
- **Fanning out** $\implies$ positive interaction ($\beta_4 > 0$)
- **Converging** $\implies$ negative interaction ($\beta_4 < 0$)

Testing Whether the Interaction Is Needed

To test whether the slopes truly differ, test $H_0 : \beta_4 = 0$ vs. $H_1 : \beta_4 \neq 0$:

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$\Longrightarrow$ This is an individual t-test, exactly the kind you already know. The only new part is what β_4 *means*.

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$\implies$ Compare models with and without the interaction using $\bar{R}^2$. If it rises, the interaction contributes to fit beyond what you would expect from chance.

Outline

Common Mistakes (1 of 2)

1. **Reporting β_2 as "the effect of education."**

   In a model with β_4(female $\times$ educ), β_2 is the effect of education *for the reference group only* (men, when female $= 0$).

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2. **Interpreting β_3 without the interaction.**

   β_3 is the gender wage gap *at* educ $= 0$. Since nobody has 0 years of education, this coefficient is rarely meaningful by itself. The full wage gap at educ $= e$ is $\beta_3 + \beta_4 e$.

Common Mistakes (2 of 2)

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4. **Including the interaction but dropping a constituent term.**

   If the model includes female $\times$ educ, it should also include both female and educ separately. Dropping a constituent term forces an artificial restriction.

Summary

	No interaction	With interaction
Model	$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + e$	$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 (x_2 x_3) + e$
Marginal effect of x_2	β_2 (constant)	$\beta_2 + \beta_4 x_3$ (varies)
Geometric picture	Parallel lines / planes	Non-parallel lines / twisted planes
Test for interaction		$H_0 : \beta_4 = 0$ (individual t-test)

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Takeaways:

- An interaction term lets the effect of one variable depend on the level of another
- The marginal effect formula comes from taking the partial derivative
- Always specify *at what value* of the other variable the marginal effect is evaluated
- Use adjusted R^2 (not R^2) to compare models with different numbers of regressors

Thank you!

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