Interaction Terms

When One Variable's Effect Depends on Another

Jake Anderson

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Outline

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A Question About Wages

Does the return to education differ for men and women?

Two possibilities:

1. Each extra year of education raises wages by **the same amount** regardless of gender
2. The payoff to education is **different** for men and women

These require different models. Our current tools (Topics 1–15) can only handle the first case. Today we learn how to handle the second.

Outline

The No-Interaction Model

Start with a model that allows wages to differ by gender, but forces the **same return to education**:

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{female}_i + e_i$$

For men ($\text{female} = 0$):

$$E(\text{wage}) = \beta_1 + \beta_2\,\text{educ}$$

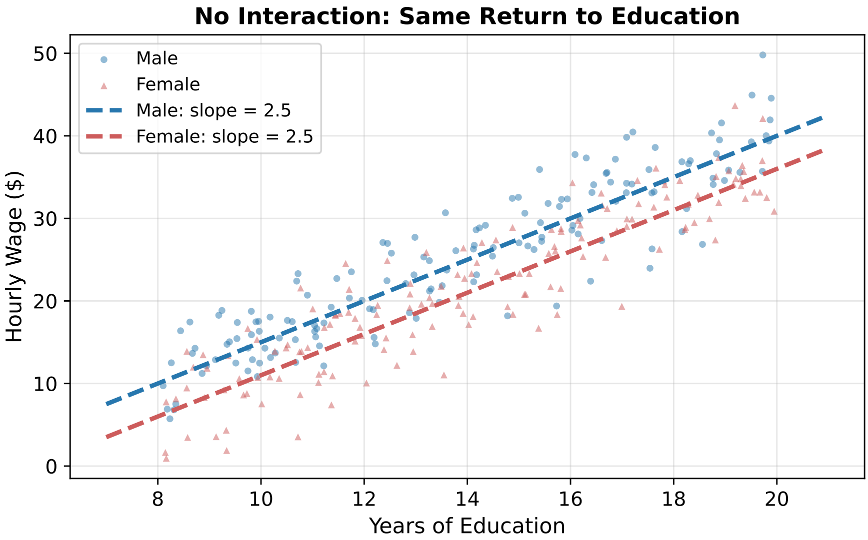
For women ($\text{female} = 1$):

$$E(\text{wage}) = (\beta_1 + \beta_3) + \beta_2\,\text{educ}$$

$\Longrightarrow$ Two lines with the **same slope** (β_2) but different intercepts.

The dummy variable female shifts the line up or down, but the return to each extra year of education is identical for both groups.

Parallel Lines: The Picture



Is This Realistic?

The parallel-lines model assumes:

- A college-educated woman benefits from education **exactly as much** as a college-educated man
- The only difference is a constant level shift

But what if the labor market rewards men's education more steeply than women's?

Or what if women with graduate degrees close the gap more than women with only high school?

$\Longrightarrow$ We need a model that allows **different slopes**, not just different intercepts.

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Adding the Interaction Term

Create a new variable: female $\times$ educ.

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{female}_i + \beta_4\,(\text{female}_i \times \text{educ}_i) + e_i$$

For men (female $= 0$):

$$E(\text{wage}) = \beta_1 + \beta_2\,\text{educ}$$

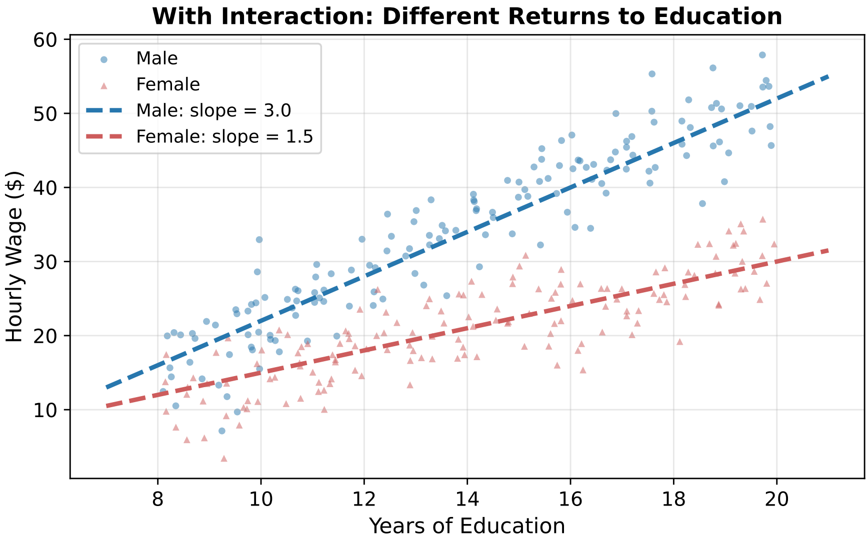
For women (female $= 1$):

$$E(\text{wage}) = \underbrace{(\beta_1 + \beta_3)}_{\text{intercept}} + \underbrace{(\beta_2 + \beta_4)}_{\text{slope}}\,\text{educ}$$

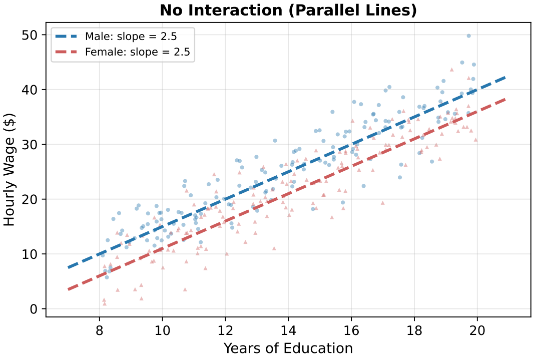
$\Longrightarrow$ Now each group has its **own intercept and its own slope.**

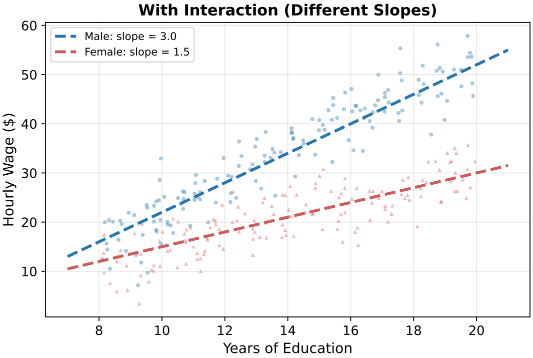
β_4 measures *how much the return to education differs* for women relative to men.

Different Slopes: The Picture



Side-by-Side Comparison





Interpreting the Interaction Coefficient

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{female}_i + \beta_4\,(\text{female}_i \times \text{educ}_i) + e_i$$

What does each coefficient measure?

- β_2: return to education **for men** (the reference group)
- β_3: wage gap at $\text{educ} = 0$ (rarely meaningful on its own)
- β_4: **difference in returns** to education, women vs. men

Interaction terms are not limited to dummy variables. The same logic applies when both variables are continuous (we will see this in detail shortly):

$$\widehat{\text{wage}} = -18.76 + 2.66\,\text{educ} + 0.24\,\text{exper} - 0.003\,(\text{educ} \times \text{exper})$$

Here $b_4 = -0.003$ tells us: a worker with one more year of education gets \$0.003/hr *less* return per year of experience.

Outline

The Marginal Effect Is No Longer a Single Number

Without interaction:

$$\text{wage} = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{female} + e \quad \Longrightarrow \quad \frac{\partial\ \mathrm{E}(\text{wage})}{\partial\,\text{educ}} = \beta_2$$

The marginal effect of education is β_2, a constant.

With interaction:

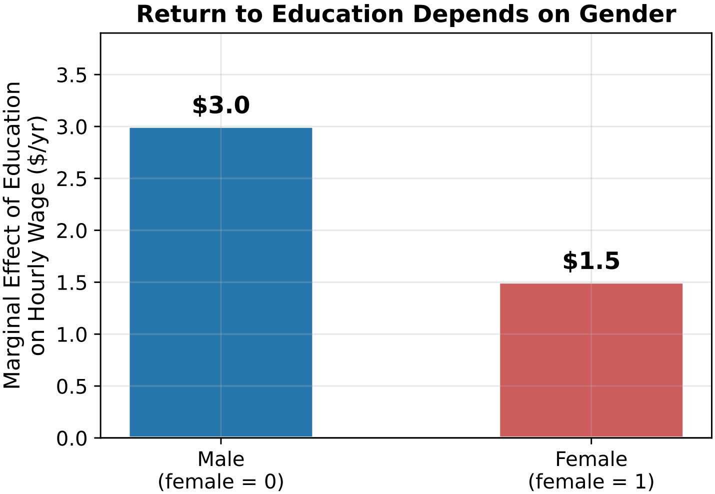
$$\text{wage} = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{female} + \beta_4(\text{female} \times \text{educ}) + e$$

$$\frac{\partial\ \mathrm{E}(\text{wage})}{\partial\,\text{educ}} = \beta_2 + \beta_4\,\text{female}$$

$\Longrightarrow$ The marginal effect of education **depends on** whether the worker is female.

You cannot report a single number for “the effect of education” without specifying who you are talking about.

Marginal Effect: The Picture



The return to education is β_2 for men and $\beta_2 + \beta_4$ for women. With $\beta_4 < 0$, women get a lower

Computing Marginal Effects: Recipe

For any interaction model $y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 (x_2 \times x_3) + e$:

Step 1: Take the partial derivative with respect to the variable of interest.

$$\frac{\partial\ \mathrm{E}(y)}{\partial\ x_2} = \beta_2 + \beta_4\ x_3 \qquad \frac{\partial\ \mathrm{E}(y)}{\partial\ x_3} = \beta_3 + \beta_4\ x_2$$

Step 2: Plug in a specific value of the *other* variable.

Step 3: Compute the numerical marginal effect.

$\implies$ Always ask: "The effect of x_2 *at what level of* x_3?"

Common choices: the sample mean, the 25th/75th percentile, or substantively meaningful values (e.g., 8 vs. 16 years of education).

Outline

When Both Variables Are Continuous

Interaction terms are not limited to dummy variables. Consider:

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{exper}_i + \beta_4\,(\text{educ}_i \times \text{exper}_i) + e_i$$

Marginal effects:

$$\frac{\partial\ \text{E(wage)}}{\partial\,\text{exper}} = \beta_3 + \beta_4\,\text{educ}$$

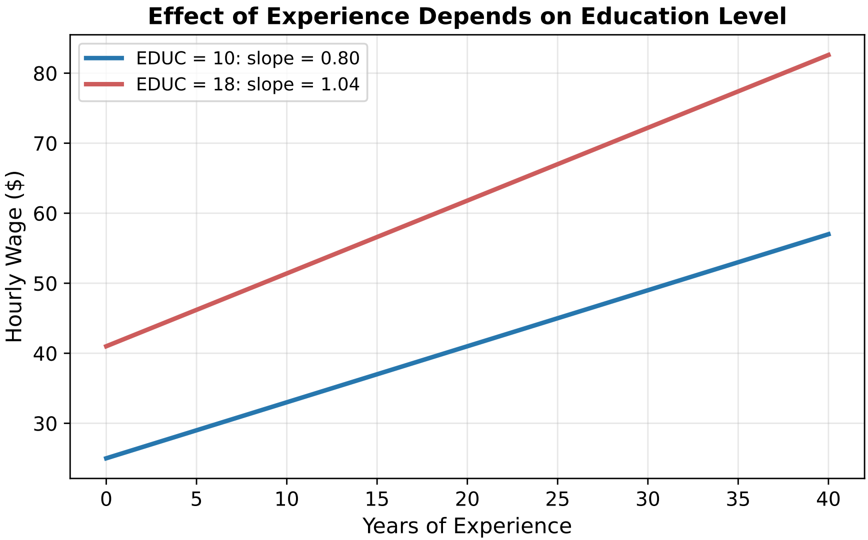
$$\frac{\partial\ \text{E(wage)}}{\partial\,\text{educ}} = \beta_2 + \beta_4\,\text{exper}$$

$\implies$ Each variable's effect depends on the level of the other variable.

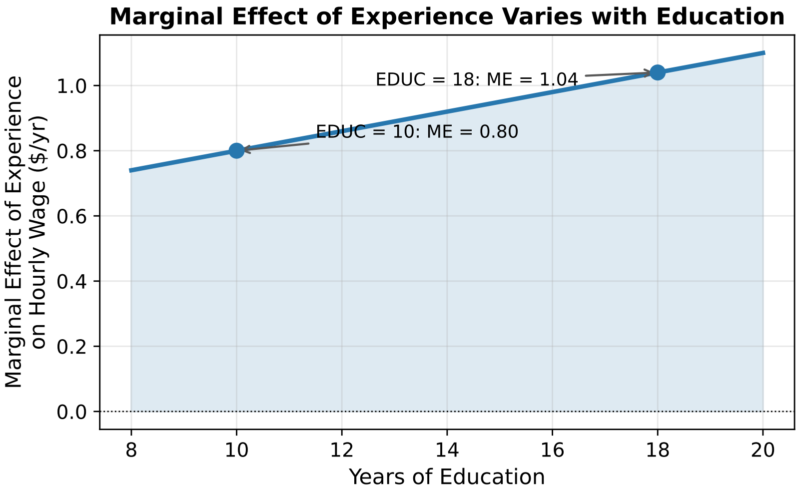
If $\beta_4 > 0$: education and experience are **complements**. More education amplifies the return to experience.

If $\beta_4 < 0$: they are **substitutes**. More education reduces the return to experience (as in the CPS data).

Visualizing the Continuous Interaction



Marginal Effect of Experience Across Education Levels



The marginal effect of experience is a **linear function** of education: $\beta_3 + \beta_4 \times \text{educ}$.

CPS Wage Equation (Textbook Example 5.15)

Using the Current Population Survey (`cps5_small`):

$$\widehat{\text{wage}} = -18.76 + 2.66\,\text{educ} + 0.24\,\text{exper} - 0.003\,(\text{educ} \times \text{exper})$$

The return to an extra year of experience:

$$\frac{\partial \widehat{\text{wage}}}{\partial\,\text{exper}} = 0.24 - 0.003 \times \text{educ}$$

	educ $= 8$	educ $= 16$
Return to experience ($/hr per year)	$0.24 - 0.003(8) = 0.22$	$0.24 - 0.003(16) = 0.19$

$\Longrightarrow$ Workers with more education get a slightly *lower* return to each additional year of experience. Education and experience act as partial substitutes.

Because $\hat{\beta}_4 < 0$, the wage–experience lines for different education levels **converge** rather than fan out.

The Log-Quadratic Wage Equation (Textbook Example 5.16)

A richer specification with $\ln(\text{wage})$ as the dependent variable:

$$\ln(\text{wage}) = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{exper} + \beta_4(\text{educ} \times \text{exper}) + \beta_5\,\text{exper}^2 + e$$

Marginal effects (approximate % changes $\times 100$):

$$\frac{\partial\, \text{E}[\ln(\text{wage})]}{\partial\, \text{exper}} = \beta_3 + \beta_4\,\text{educ} + 2\beta_5\,\text{exper}$$

$$\frac{\partial\, \text{E}[\ln(\text{wage})]}{\partial\, \text{educ}} = \beta_2 + \beta_4\,\text{exper}$$

$\Longrightarrow$ The interaction term and the quadratic term *both* make marginal effects vary. You need to specify values of *all* interacting variables to compute a marginal effect.

The textbook (Table 5.4) reports that the return to education is 2.53 percentage points lower for someone with 20 years of experience vs. 0.

Outline

Plotting Interaction Effects

To plot an interaction, fix the "other" variable at specific values and graph the predicted outcome against the variable of interest.

For $y = \beta_1 + \beta_2\,\text{educ} + \beta_3\,\text{exper} + \beta_4(\text{educ} \times \text{exper}) + e$:

1. Choose representative values of educ (e.g., 10, 14, 18)
2. For each value, plot $\widehat{y}$ against exper
3. Each value of educ gives a line with slope $\hat{\beta}_3 + \hat{\beta}_4 \times \text{educ}$

What to look for:

- **Parallel lines** $\implies$ no interaction ($\beta_4 \approx 0$)
- **Fanning out** $\implies$ positive interaction ($\beta_4 > 0$)
- **Converging** $\implies$ negative interaction ($\beta_4 < 0$)

Testing Whether the Interaction Is Needed

To test whether the slopes truly differ, test $H_0 : \beta_4 = 0$ vs. $H_1 : \beta_4 \neq 0$:

$$t = \frac{b_4}{\text{se}(b_4)} \sim t_{(N-K)}$$

If we reject H_0: The interaction is statistically significant. The marginal effect of one variable genuinely depends on the other.

If we fail to reject H_0: The parallel-lines (or constant-marginal-effect) model may be adequate.

$\Longrightarrow$ This is an individual t-test, exactly the kind you already know. The only new part is what β_4 *means*.

Outline

Comparing Models: Use $\bar{R}^2$

Recall from Topic 13: R^2 can never fall when you add a regressor, so it cannot tell you whether the interaction term genuinely improves the model.

Use the **adjusted** R^2 instead:

$$\bar{R}^2 = 1 - \frac{SSE/(N-K)}{SST/(N-1)}$$

It penalizes for each additional parameter: $\bar{R}^2$ rises only when a new variable reduces SSE enough to offset the lost degree of freedom.

$\Longrightarrow$ Compare models with and without the interaction using $\bar{R}^2$. If it rises, the interaction contributes to fit beyond what you would expect from chance.

Outline

Common Mistakes (1 of 2)

1. **Reporting β_2 as "the effect of education."**

   In a model with β_4(female $\times$ educ), β_2 is the effect of education *for the reference group only* (men, when female $= 0$).

2. **Interpreting β_3 without the interaction.**

   β_3 is the gender wage gap *at* educ $= 0$. Since nobody has 0 years of education, this coefficient is rarely meaningful by itself. The full wage gap at educ $= e$ is $\beta_3 + \beta_4 e$.

Common Mistakes (2 of 2)

3. **Forgetting that marginal effects vary.**

   You must evaluate the marginal effect at a specific value of the other variable. Writing "the marginal effect of education is β_2" is incomplete.

4. **Including the interaction but dropping a constituent term.**

   If the model includes female $\times$ educ, it should also include both female and educ separately. Dropping a constituent term forces an artificial restriction.

Summary

	No interaction	With interaction
Model	$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + e$	$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 (x_2 x_3) + e$
Marginal effect of x_2	β_2 (constant)	$\beta_2 + \beta_4 x_3$ (varies)
Geometric picture	Parallel lines / planes	Non-parallel lines / twisted planes
Test for interaction		$H_0 : \beta_4 = 0$ (individual t-test)

Takeaways:

- An interaction term lets the effect of one variable depend on the level of another
- The marginal effect formula comes from taking the partial derivative
- Always specify *at what value* of the other variable the marginal effect is evaluated
- Use adjusted R^2 (not R^2) to compare models with different numbers of regressors

Thank you!

jakeanderson@g.ucla.edu