Model Specification, Multicollinearity, and Model Selection

What Happens When You Leave Something Out, Put Too Much In, or Can't Tell Them Apart

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Outline

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The Scenario

You run a wage regression:

$$\text{wage}_i = \beta_1 + \beta_2 \, \text{educ}_i + e_i$$

You get $\hat{\beta}_2 = 2.3$: each extra year of education raises wages by \$2.30/hr.

But **ability** also affects wages, and more able people tend to get more education.

Question: Is \$2.30 the true return to education, or is some of it actually the return to ability?

The True Model vs. What We Estimated

True model (what we should estimate):

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3 \underbrace{\text{ability}_i}_{\text{omitted!}} + e_i$$

Estimated model (what we actually run):

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \underbrace{(\beta_3\,\text{ability}_i + e_i)}_{v_i}$$

The omitted variable gets absorbed into the error term v_i.

If $\text{Cov}(\text{educ}, \text{ability}) \neq 0$, then $\text{Cov}(\text{educ}, v) \neq 0$.

$\implies$ The exogeneity assumption $\text{Cov}(x, e) = 0$ is violated. OLS is **biased**.

The OVB Formula

How much bias? There is an exact formula. Let:

- b_2^* = the OLS estimate from the misspecified (short) regression
- β_2 = the true coefficient on education

Omitted variable bias:

$$\underbrace{E(b_2^*)}_{\text{what we get}} = \underbrace{\beta_2}_{\text{true effect}} + \underbrace{\beta_3}_{\substack{\text{effect of} \\ \text{ability on} \\ \text{wage}}} \times \underbrace{\frac{\text{Cov}(\text{educ}, \text{ability})}{\text{Var}(\text{educ})}}_{\substack{\text{slope from regressing} \\ \text{ability on educ}}}$$

In shorthand: $\text{bias}(b_2^*) = \beta_3 \times \hat{\delta}_1$

where $\hat{\delta}_1$ is the OLS slope from the **auxiliary regression** $\text{ability}_i = \delta_0 + \delta_1 \, \text{educ}_i + v_i$.

Signing the Bias

$$\text{bias}(b_2^*) = \underbrace{\beta_3}_{\text{sign?}} \times \underbrace{\hat{\delta}_1}_{\text{sign?}}$$

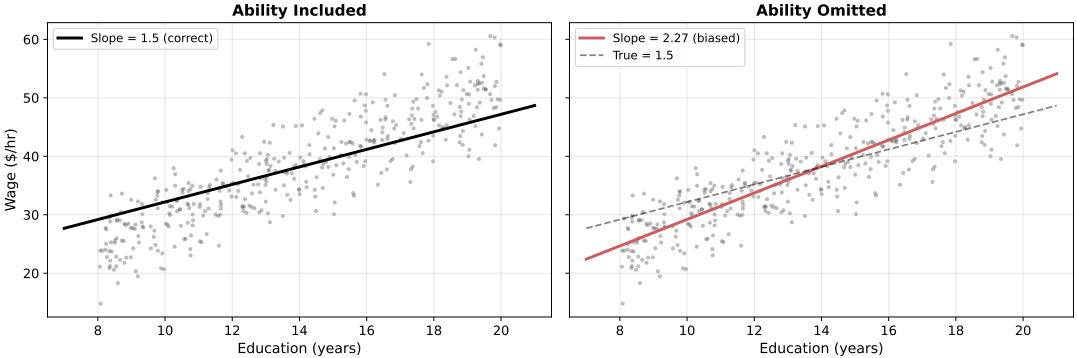
For the wage regression:

- $\beta_3 > 0$: higher ability $\implies$ higher wages
- $\hat{\delta}_1 > 0$: more able people get more education

$\implies$ Bias is **positive**. The estimated return to education is **too large** because OLS attributes some of ability's effect to education.

	$\hat{\delta}_1 > 0$ (positive correlation)	$\hat{\delta}_1 < 0$ (negative correlation)
$\beta_3 > 0$ (OV helps y)	Upward bias	Downward bias
$\beta_3 < 0$ (OV hurts y)	Downward bias	Upward bias

Visualizing OVB



The biased regression line is steeper: it picks up the indirect effect of ability through education.

OVB Is Not a Small-Sample Problem

OVB does **not** go away with more data. The inconsistency formula:

$$b_2^* \xrightarrow{p} \beta_2 + \beta_3 \frac{\text{Cov}(\text{educ}, \text{ability})}{\text{Var}(\text{educ})}$$

As $N \to \infty$, the estimator converges to the **wrong value**.

$\Longrightarrow$ This is not about imprecision (which N fixes). The model is wrong, and no amount of data corrects a wrong model.

Two conditions that eliminate OVB:

1. $\beta_3 = 0$ (the omitted variable doesn't affect y)
2. $\text{Cov}(x, z) = 0$ (the omitted variable is uncorrelated with the included regressor)

$\Longrightarrow$ An omitted variable is only a problem when **both** conditions fail.

OVB: Wage Regression Example

Using wage data ($N = 1{,}057$, Koop–Tobias):

	Without ability	With ability (SCORE)
Return to education	7.3%	5.9%

Including an ability proxy reduces the education coefficient by 1.4 percentage points.

$\Longrightarrow$ Without the control, the estimated return to education was inflated by about 24% (relative to the controlled estimate).

Even this is only a partial fix: SCORE is a **proxy** for ability, not ability itself. Some bias likely remains.

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The Opposite Mistake

What if, instead of leaving out a relevant variable, you include a variable that **doesn't belong**?

True model: $\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{ability}_i + e_i$

You estimate:

$$\text{wage}_i = \beta_1 + \beta_2\,\text{educ}_i + \beta_3\,\text{ability}_i + \beta_4\,\underbrace{\text{shoe size}_i}_{\beta_4=0} + e_i$$

Since $\beta_4 = 0$ in the true model, shoe size is **irrelevant**.

What happens to the other estimates?

Irrelevant Variables: Unbiased but Inefficient

Good news: OLS estimates of β_1, β_2, β_3 remain **unbiased**.

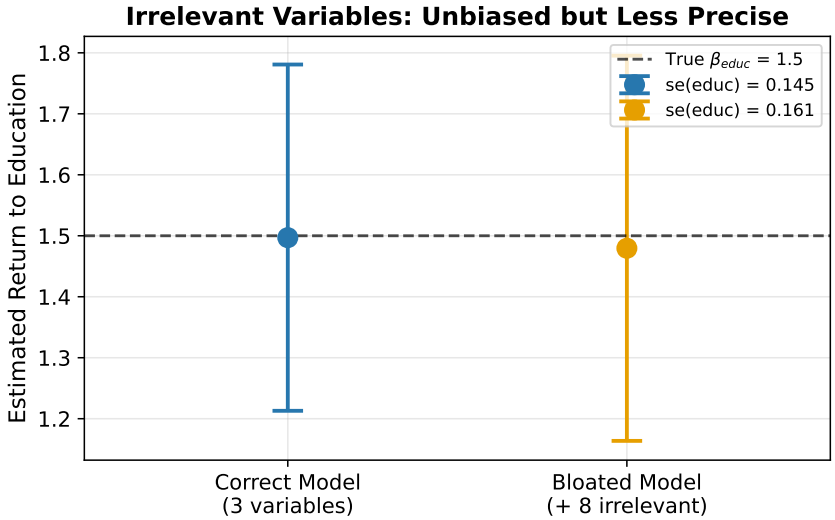
Bad news: Their standard errors **increase**.

Why? The variance formula for b_k includes a term $1/(1 - R_k^2)$ from the auxiliary regression of x_k on all other regressors. Adding shoe size to the regressor list increases R_k^2 (even slightly), which inflates $\text{Var}(b_k)$.

$\Longrightarrow$ Wider confidence intervals, larger p-values, less statistical power.

You might fail to reject $H_0 : \beta_2 = 0$ not because education doesn't affect wages, but because you wasted degrees of freedom on shoe size.

Visualizing the Precision Loss



OVB vs. Irrelevant Variables: The Tradeoff

	Omit relevant variable	**Include irrelevant variable**
Bias?	Yes (doesn't vanish)	No
Variance?	Lower	Higher
Consistency?	No	Yes

$\Longrightarrow$ Given a choice between the two mistakes, including an irrelevant variable is the **lesser evil**. You pay with precision, not with bias.

But this doesn't mean "throw everything in." More variables = less precision = wider intervals. The goal is a **well-specified** model.

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Can We Test Whether Our Model Is Wrong?

The **RESET** (REgression Specification Error Test) checks for functional form misspecification and omitted variables.

Idea: If the model is correctly specified, then nonlinear functions of the fitted values $\hat{y}$ should not have additional explanatory power. Since $\hat{y}$ is a linear combination of the x's, $\hat{y}^2$ and $\hat{y}^3$ are polynomial functions of the regressors. They approximate many different nonlinear functional forms without specifying one.

Procedure:

1. Estimate your model: $y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + e$
2. Compute fitted values $\hat{y}$
3. Estimate the augmented model:

$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \gamma_1 \hat{y}^2 + \gamma_2 \hat{y}^3 + e$$

4. Test $H_0 : \gamma_1 = \gamma_2 = 0$ using an F-test

Interpreting RESET

Reject H_0: The model is misspecified.

- Could be omitted variables, wrong functional form, or both
- RESET does not tell you *what* is wrong, only that *something* is

Fail to reject H_0: Not conclusive evidence that the model is correct.

- RESET has limited power against certain alternatives
- A model can pass RESET and still be misspecified

$\Longrightarrow$ RESET is a **general-purpose** diagnostic, not a test for a specific alternative.

RESET: Family Income Example

Applying RESET to the family income equation $\ln(\text{FAMINC}) = \beta_1 + \beta_2\,\text{HEDU} + \beta_3\,\text{WEDU} + e$:

Specification	RESET p-value	Verdict
Both HEDU and WEDU	0.75	Passes
WEDU omitted	0.04	**Rejects**
Both + children (KL6)	0.49	Passes
Both + irrelevant vars	0.43	Passes

RESET detects the omitted variable (WEDU) but does not flag irrelevant variables.

$\Longrightarrow$ Useful as a diagnostic, but not a substitute for careful model building.

We have checked whether the model form is correct. Next: what if the model is right but the data makes estimation difficult?

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A Different Problem

New scenario: You include both education and experience in the wage regression:

$$\text{wage}_i = \beta_1 + \beta_2 \, \text{educ}_i + \beta_3 \, \text{exper}_i + e_i$$

Both variables belong in the model. Neither is irrelevant.

But in your data, education and experience are **highly correlated**: people who stayed in school longer have fewer years of work experience.

Question: What happens to your estimates when the regressors move together?

What Is Collinearity?

Collinearity (or multicollinearity): explanatory variables are highly correlated with each other.

Exact collinearity ($r = \pm 1$): one regressor is a perfect linear function of another. OLS cannot compute unique estimates.

Near collinearity ($|r|$ close to 1): OLS is defined but the estimates are very imprecise.

Not a violation of assumptions. The Gauss–Markov theorem still holds. OLS is still BLUE.

$\implies$ The problem is not bias. The problem is that "Best Linear Unbiased" can still mean **very imprecise**.

The Variance Formula: Where Collinearity Hurts

For two regressors, the variance of b_2 is:

$$\text{Var}(b_2) = \frac{\sigma^2}{\sum_{i=1}^{N}(x_{i2} - \bar{x}_2)^2 \cdot \underbrace{(1 - r_{23}^2)}_{\text{collinearity factor}}}$$

- If $r_{23} = 0$: no collinearity, $(1 - r_{23}^2) = 1$, variance is at its minimum
- If $r_{23} = 0.9$: $(1 - r_{23}^2) = 0.19$, variance is **5.3 times larger**
- If $r_{23} = 0.99$: $(1 - r_{23}^2) = 0.02$, variance is **50 times larger**

$\Longrightarrow$ High correlation between regressors inflates the variance of the coefficient estimates.

The Variance Inflation Factor (VIF)

With more than two regressors, the **auxiliary regression** replaces the simple correlation.

Regress x_2 on *all other* regressors: $x_{i2} = \delta_0 + \delta_1 x_{i3} + \cdots + \delta_{K-1} x_{iK} + v_i$

Let R_2^2 = the R^2 from this auxiliary regression. Then:

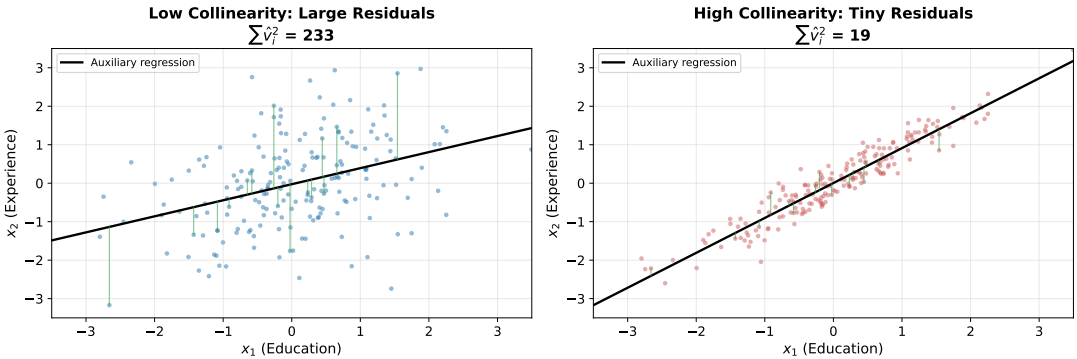
$$\text{Var}(b_2) = \frac{\sigma^2}{\sum (x_{i2} - \bar{x}_2)^2} \cdot \underbrace{\frac{1}{1 - R_2^2}}_{\text{VIF}}$$

Variance Inflation Factor:

$$\text{VIF} = \frac{1}{1 - R_2^2}$$

- VIF = 1: no collinearity (baseline variance)
- VIF = 10: variance is 10 times the baseline ($R_2^2 = 0.9$)
- Rule of thumb: VIF > 10 signals problematic collinearity

Why Does This Happen? The Auxiliary Regression View



OLS estimates β_2 using only the variation in x_2 that is **not explained by the other regressors**. When collinearity is high, very little unique variation remains.

Symptoms of Collinearity

How do you know collinearity is affecting your results?

1. **High R^2, insignificant t-statistics:** The model explains a lot, but you can't say which variables are doing the work
2. **Large standard errors:** Confidence intervals for individual coefficients are very wide
3. **Unstable coefficients:** Adding or removing one observation (or one variable) changes the estimates substantially
4. **Joint significance but individual insignificance:** The F-test rejects $H_0 : \beta_2 = \beta_3 = 0$ even though neither t-test rejects individually

$\implies$ If you see a high overall F but low individual t's, check for collinearity before dropping variables.

Rice Production Example

Philippine rice farmers ($N = 344$, 1994 data):

$$\ln(\text{PROD}) = \beta_1 + \beta_2 \ln(\text{AREA}) + \beta_3 \ln(\text{LABOR}) + \beta_4 \ln(\text{FERT}) + e$$

Variable	**Coeff**	***p*-value**	**VIF**	
ln(AREA)	0.339	0.198	9.2	insignificant
ln(LABOR)	0.175	0.465	17.9	insignificant
ln(FERT)	0.279	0.003	7.7	significant
Joint test: $H_0 : \beta_2 = \beta_3 = 0$			$p = 0.0021$	
Overall R^2			0.875	

$\Longrightarrow$ Area and labor are individually insignificant, but jointly significant ($p = 0.002$). Dropping them would introduce OVB.

What Can You Do About Collinearity?

Option 1: Get more data

- More observations can help if the new data provide independent variation
- Combining 1993 and 1994 rice data: VIFs drop, all coefficients become significant

Option 2: Use nonsample information

- Impose restrictions from economic theory (e.g., constant returns to scale: $\beta_2 + \beta_3 + \beta_4 = 1$)
- This reduces the number of free parameters and narrows confidence intervals
- But if the restriction is wrong, you introduce bias

What you should not do:

- Do not drop a variable just because its t-statistic is insignificant
- If the variable belongs in the model theoretically, removing it causes OVB

Outline

Choosing Among Competing Models

You have several candidate models for y. How do you choose?

R^2 **is not enough.** It always increases when you add variables, even irrelevant ones.

We need criteria that **penalize complexity**: reward a good fit, but punish you for using too many parameters.

Three tools:

1. Adjusted R^2 ($\bar{R}^2$)
2. Akaike Information Criterion (AIC)
3. Schwarz / Bayesian Information Criterion (SC / BIC)

Adjusted R^2

$$\bar{R}^2 = 1 - \frac{SSE/(N-K)}{SST/(N-1)}$$

Unlike R^2, the adjusted R^2 does **not** automatically increase when a variable is added.

When does it increase? When the added variable's $|t| > 1$.

- $|t| > 1$: the variable reduces $SSE/(N-K)$ enough to compensate for the lost degree of freedom
- $|t| < 1$: the variable does not reduce $SSE/(N-K)$ enough; $\bar{R}^2$ falls

Limitation: The threshold $|t| > 1$ corresponds to a significance level of about 32%, which is much weaker than the usual 5%. Adjusted R^2 is too generous with extra variables.

AIC and BIC

Akaike Information Criterion:

$$\text{AIC} = \ln\left(\frac{SSE}{N}\right) + \frac{2K}{N}$$

Schwarz / Bayesian Information Criterion:

$$\text{BIC} = \ln\left(\frac{SSE}{N}\right) + \frac{K\ln(N)}{N}$$

Both have the form: $\underbrace{\ln(SSE/N)}_{\text{goodness of fit}} + \underbrace{\text{penalty}(K, N)}_{\text{complexity cost}}$

- **Smaller is better** for both
- Adding a variable decreases *SSE* (improving the first term) but increases the penalty (worsening the second term)
- The preferred model balances fit against parsimony

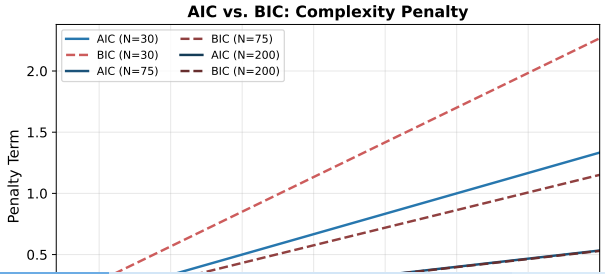
AIC vs. BIC: Which Penalizes More?

Compare the penalty terms for one additional parameter:

- AIC penalty per parameter: $\frac{2}{N}$
- BIC penalty per parameter: $\frac{\ln(N)}{N}$

BIC penalizes more heavily whenever $\ln(N) > 2$, i.e., $N > e^2 \approx 7.4$.

$\Longrightarrow$ For any practical sample size ($N \geq 8$), BIC is **more conservative** than AIC.



Using Model Selection Criteria: House Price Example

Baton Rouge houses ($N = 800$ estimation, 100 held out):

Model	Variables	$\bar{R}^2$	AIC	BIC	RMSE
1	SQFT, AGE	0.463	–	–	0.327
2	SQFT, AGE, AGE^2	**0.476**	–	–	0.322
3	SQFT, AGE^2	0.473	–	–	0.326

Model 2 (with the AGE^2 term) is favored by $\bar{R}^2$, AIC, and BIC.

Important restriction: You can only compare models with the **same dependent variable**. You cannot use AIC/BIC to compare a model for y against a model for $\ln(y)$.

Outline

What To Take Away

1. **Omit a relevant variable** $\implies$ biased and inconsistent estimates; bias direction = sign$(\beta_3)\times$ sign$(\text{Cov}(x, z))$
2. **Include an irrelevant variable** $\implies$ unbiased but larger standard errors (less precision)
3. **RESET** tests for misspecification by adding $\hat{y}^2, \hat{y}^3$; rejection signals a problem, but passing is not a guarantee
4. **Multicollinearity** inflates variances but does not bias estimates; diagnose with VIF > 10; do not drop theoretically justified variables
5. **Model selection:** use adjusted R^2, AIC, or BIC (not R^2) to compare models; BIC penalizes complexity more heavily than AIC

Decision Checklist

When evaluating a regression model, ask:

1. Did I leave out a variable that affects y **and** correlates with x? $\implies$ OVB
2. Did I include variables that have no business being in the model? $\implies$ Inflated standard errors
3. Does RESET reject? $\implies$ Something is wrong with the functional form or variable list
4. Are any VIFs above 10? $\implies$ Collinearity; do not confuse imprecision with irrelevance
5. Am I comparing models with the same y? $\implies$ Use AIC/BIC; lower is better

Thank you!
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