Treatment Effects and Difference-in-Differences

Everything You've Learned, Applied to Policy

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Outline

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A State Raised Its Minimum Wage. Did Employment Fall?

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We cannot observe what *would have happened* in NJ without the increase. This unobserved scenario is the **counterfactual**.

$\implies$ We need a strategy that isolates the policy effect from everything else changing at the same time.

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$\Longrightarrow$ We need a **control group** that experienced the same time trends but *not* the policy change.

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Treatment and Control Groups

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$\implies$ β_2 is the **treatment effect**: the average difference in y between treated and control.

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For $\mathrm{E}(b_2) = \beta_2$, we need $\mathrm{E}(\bar{e}_1) = \mathrm{E}(\bar{e}_0)$: all other factors must be equal on average across groups.

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Here $\bar{e}_1$ is the average unobserved health factor for those who visit and $\bar{e}_0$ is the average for those who do not. When $E(\bar{e}_1 - \bar{e}_0) \neq 0$, the simple difference estimator is biased. This is **selection bias.**

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$\Longrightarrow$ Random assignment eliminates selection bias by ensuring $E(\bar{e}_1) = E(\bar{e}_0)$.

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But we still have the time-trend problem. PA employment also changed between February and November. We need a method that accounts for this.

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First difference (within each group over time):

- Control change: $\bar{y}_{C,\text{After}} - \bar{y}_{C,\text{Before}}$ (time trend only)
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$\Longrightarrow$ The time trend cancels, leaving only the **treatment effect.**

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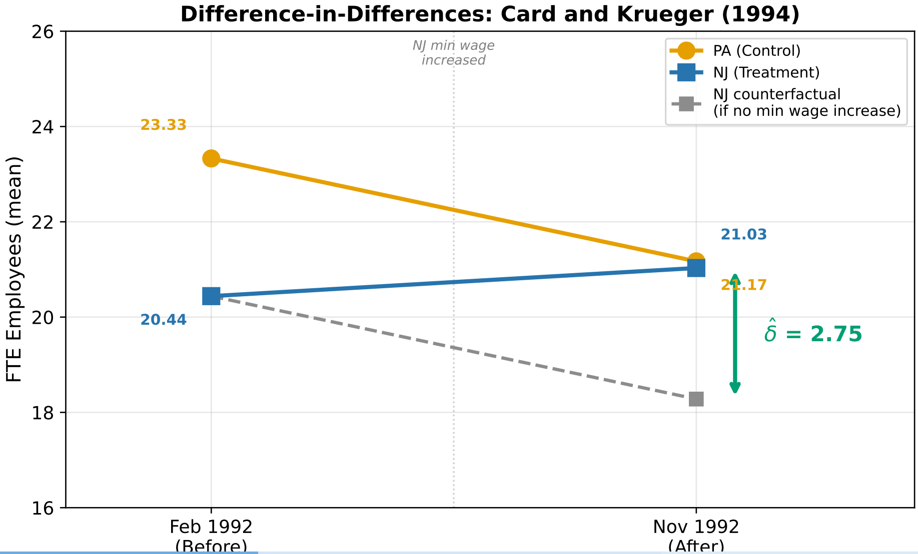
Employment in NJ *increased* by about 2.75 FTE relative to PA.

Visualizing DiD



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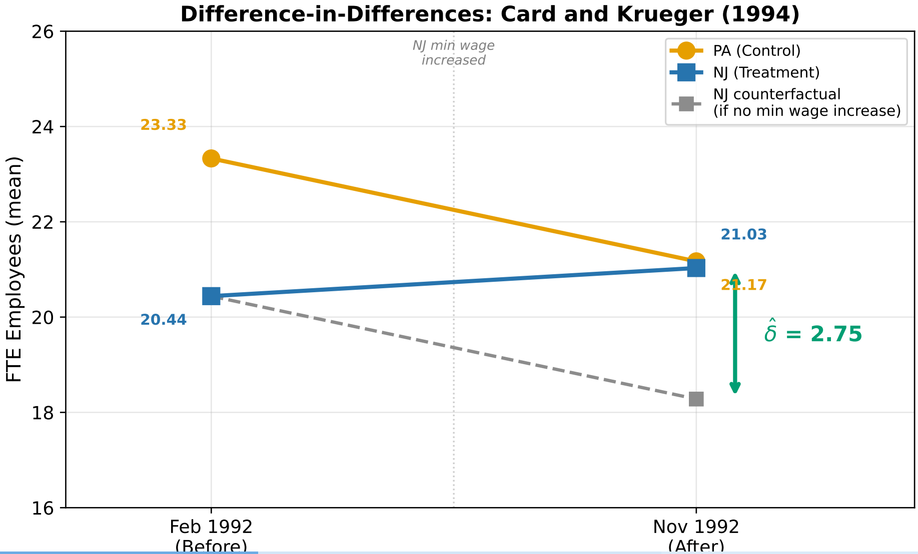

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From the 2×2 Table to a Regression

The DiD estimate can be obtained from a single regression:

$$y_{it} = \beta_1 + \beta_2 \, \text{NJ}_i + \beta_3 \, \text{After}_t + \delta \, (\text{NJ}_i \times \text{After}_t) + e_{it}$$

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This uses everything you already know:

- NJ_i: an **indicator variable** (Topic 19)
- After_t: another indicator variable
- $\text{NJ}_i \times \text{After}_t$: an **interaction term** (Topic 16)

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$\Longrightarrow$ DiD is not a new technique. It is a regression with two dummies and their interaction.

What Each Coefficient Means

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Write out the regression function for each cell:

	$\text{After} = 0$	$\text{After} = 1$
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- $\beta_1 = 23.33$: PA mean before (reference cell)
- $\beta_2 = -2.89$: NJ was 2.89 FTE *below* PA before the policy
- $\beta_3 = -2.17$: PA employment fell by 2.17 (time trend)
- $\delta = 2.75$: the **treatment effect** (DiD estimator)

Verifying: δ Recovers the DiD Formula

From the table of regression functions:

NJ change:

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You can test $H_0 : \delta = 0$ with a standard t-test. In Card and Krueger's data: $t \approx 1.63$, $p \approx 0.10$ (two-sided). We fail to reject at the 5% level. A two-sided test is appropriate here because the minimum wage could plausibly raise *or* lower employment.

Adding Controls

Just like any regression, we can add control variables:

$$y_{it} = \beta_1 + \beta_2 \, \text{NJ}_i + \beta_3 \, \text{After}_t + \delta \, (\text{NJ}_i \times \text{After}_t) + \gamma_1 \text{KFC}_i + \gamma_2 \text{Roys}_i + \gamma_3 \text{Wendys}_i + e_{it}$$

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$\Longrightarrow$ Adding controls can improve precision and test robustness, just as in multiple regression (Topic 13).

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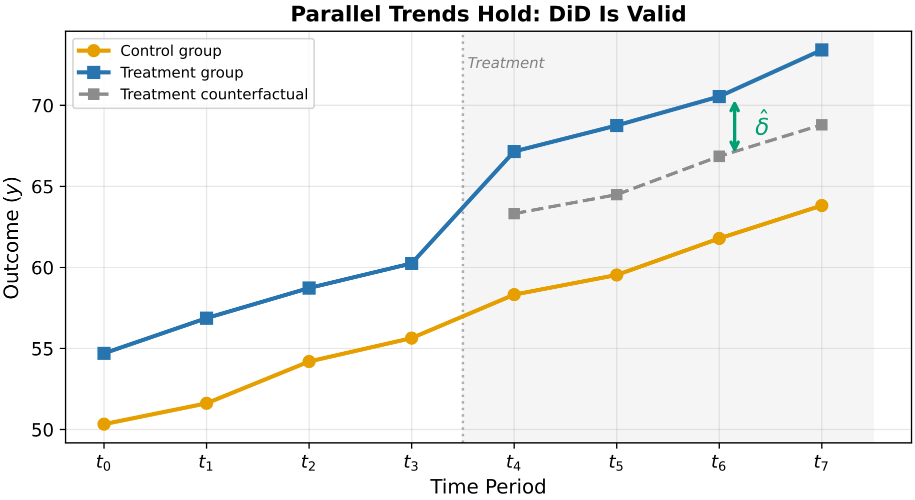
In words: the treatment group's trend (absent treatment) would have matched the control group's trend.

The levels can differ. The trends must be the same.

When Parallel Trends Hold



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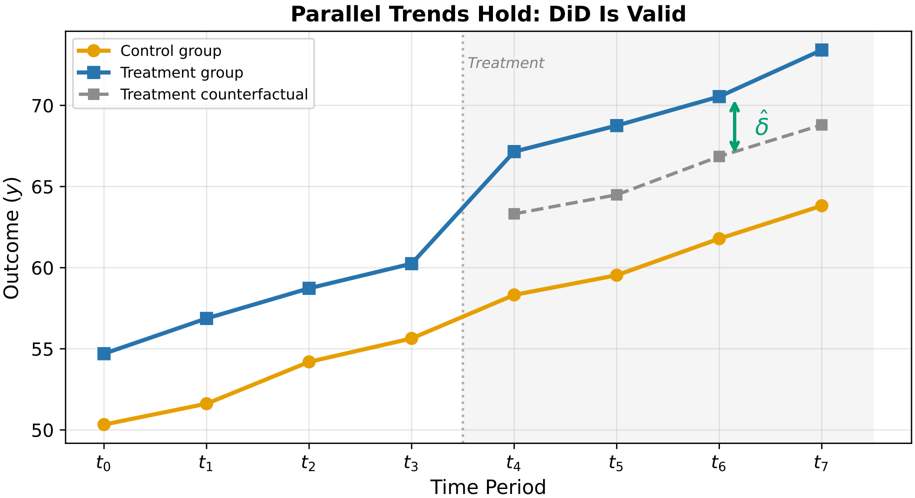
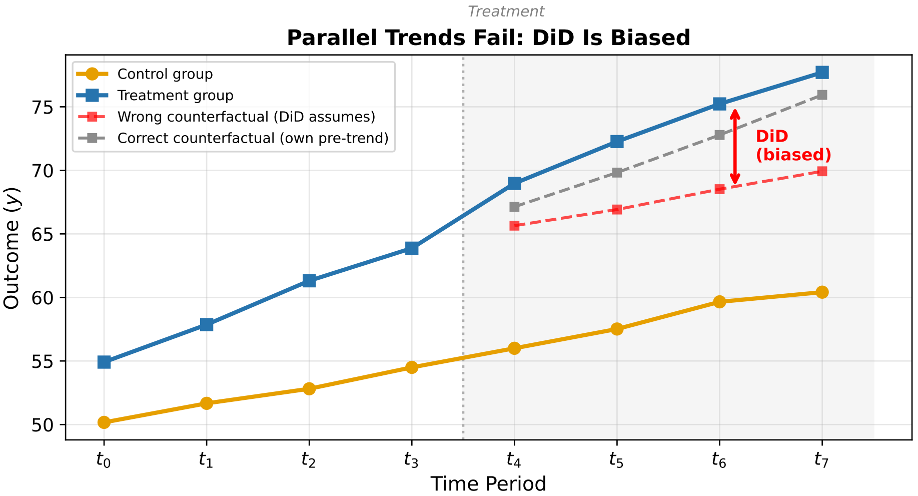


Before treatment, both groups follow the same trend (the lines are parallel). After treatment, the

When Parallel Trends Fail



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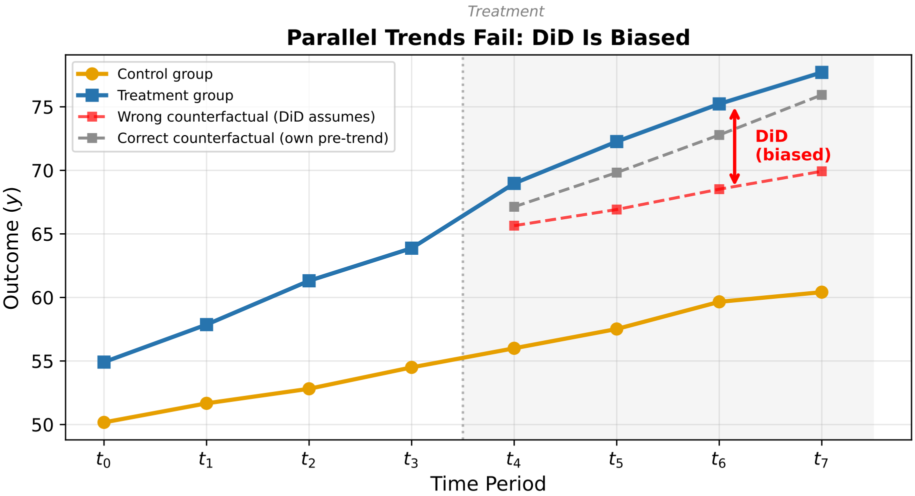

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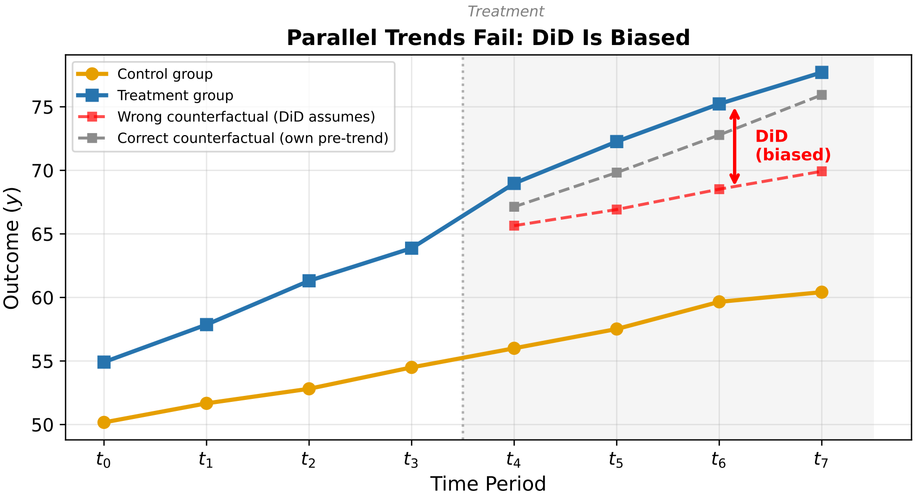

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In Card and Krueger's case: NJ and PA are neighboring states with similar fast-food markets. The assumption is plausible, though debated.

Outline

Using Panel Data: First Differencing

Card and Krueger observed 384 restaurants in *both* periods. With panel data, we can control for unobserved restaurant-specific characteristics:

$$\text{FTE}_{it} = \beta_1 + \beta_2\, \text{NJ}_i + \beta_3\, D_t + \delta\, (\text{NJ}_i \times D_t) + c_i + e_{it}$$

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$\Longrightarrow$ This is the same idea as the fixed effects estimator (previewed here, covered fully in Econ 104).

Outline

DiD Uses Your Entire Toolkit

Tool	Topic	Role in DiD
Indicator variables	19	NJ_i and After_t
Interaction terms	16	$\text{NJ}_i \times \text{After}_t$
Interpreting coefficients	14	What $\beta_1, \beta_2, \beta_3, \delta$ mean
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$\Longrightarrow$ Policy evaluation is not a separate topic. It is the application of every regression concept you have learned this quarter.

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5. DiD requires **parallel trends**: absent treatment, both groups would have changed the same way. Check plausibility with pre-treatment data.
6. Panel data lets you control for unit-specific fixed effects by first-differencing.

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$\Longrightarrow$ You now have the foundation for all of it.

Thank you!

Good luck on the final.

jakeanderson@g.ucla.edu