LPM vs. Logit/Probit

Modeling Binary Outcomes Without Impossible Predictions

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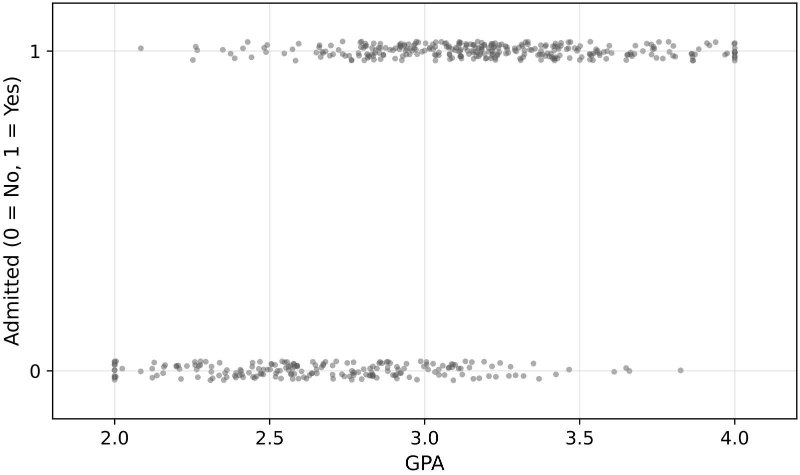
Outline

The Data

A university admissions office records **GPA** and **admission outcome** (admit/reject) for 500 applicants.



The outcome is binary: 0 (rejected) or 1 (admitted). How do we model the probability of admission?

Natural Instinct: Run OLS

The simplest approach: regress the 0/1 outcome on GPA, just like any other regression.

This is the **Linear Probability Model** (LPM):

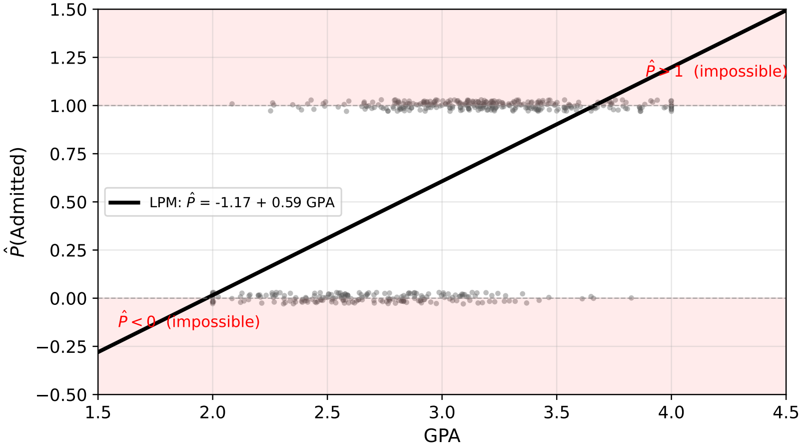
$$P(\text{Admit}_i = 1 \mid \text{GPA}_i) = \beta_0 + \beta_1 \, \text{GPA}_i$$

The coefficients have a direct interpretation:

- β_1 = change in the *probability of admission* for a one-unit increase in GPA

Sounds reasonable. Let's see what happens.

LPM Fit



$\hat{P}(\text{Admit}) = -1.17 + 0.59 \cdot \text{GPA}$. At GPA $= 4.0$: $\hat{P} = 1.20$. At GPA $= 2.0$: $\hat{P} = 0.02$.

$\Longrightarrow$ Probabilities **must** lie in $[0, 1]$. A straight line cannot respect this constraint.

Problem 1: Impossible Predictions

A probability model should produce $\hat{P} \in [0, 1]$ for all observations. The LPM violates this.

For any linear function $\hat{P} = \beta_0 + \beta_1 x$:

- If x is large enough $\implies \hat{P} > 1$
- If x is small enough $\implies \hat{P} < 0$

In our data: applicants with GPA above ≈ 3.7 get predicted probabilities exceeding 1.

$\implies$ The LPM is a line forced through inherently nonlinear data. It works in the middle but fails in the tails.

Problem 2: Constant Marginal Effects

The LPM says: each additional GPA point increases admission probability by **0.59**, regardless of where you start.

- Going from GPA 2.0 to 3.0: $+0.59$ probability
- Going from GPA 3.0 to 4.0: $+0.59$ probability

Is that realistic?

No. Consider the S-shaped relationship we expect:

- Near the middle of the GPA range, the probability is changing rapidly (steep part of the curve)
- At the extremes, the probability is near 0 or near 1, so an extra GPA point makes little difference (flat parts of the curve)

$\Longrightarrow$ Marginal effects should be **largest near the midpoint** and diminish in the tails, not constant everywhere.

Problem 3: Heteroskedastic Errors

When y can only be 0 or 1, its variance is the variance of a Bernoulli random variable:

$$\text{Var}(y_i \mid x_i) = P(x_i)\big(1 - P(x_i)\big)$$

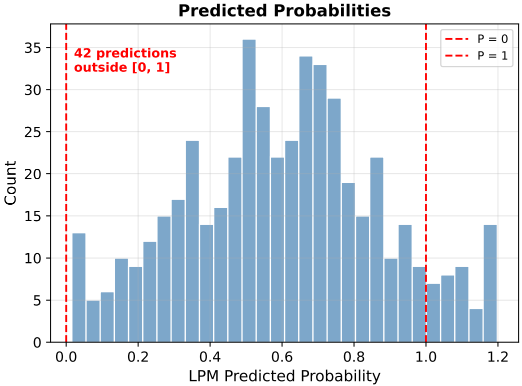
This varies with x by construction $\implies$ **heteroskedasticity is guaranteed**.

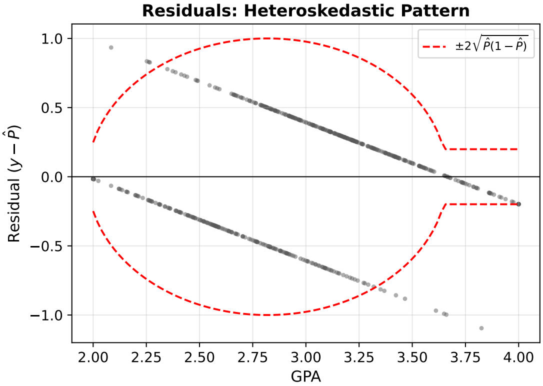
Consequences:

- OLS coefficients are still **unbiased**
- But OLS standard errors are **wrong** (too small or too large)
- Hypothesis tests and confidence intervals are unreliable

This problem is fixable: robust standard errors correct the SEs. But the impossible predictions and constant marginal effects remain.

LPM Problems: Visualized





Left: some predictions fall outside $[0, 1]$. Right: residuals fan out, confirming heteroskedasticity.

The Root Cause

All three LPM problems share one structural mismatch:

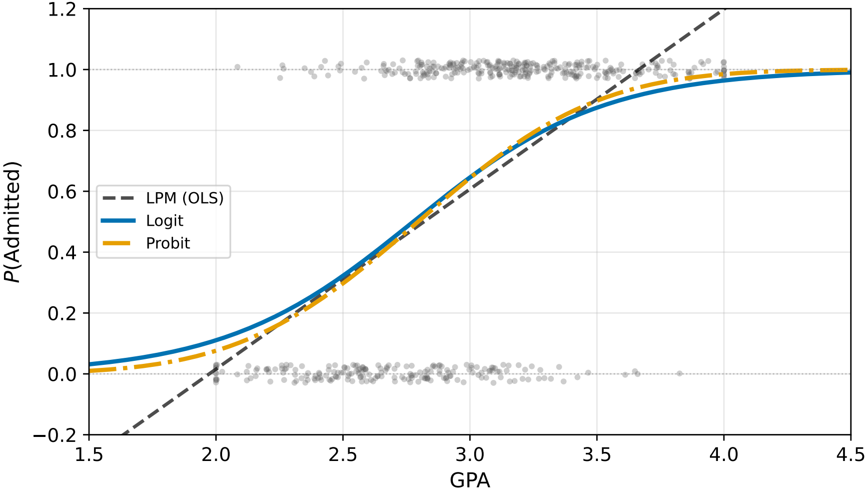
A straight line has no bounds, but a probability does.

- Problems 1 and 2 are two faces of this mismatch:
  - **Out of bounds** $\implies$ the line overshoots $[0, 1]$
  - **Constant slope** $\implies$ the line cannot flatten as it approaches 0 or 1
- Problem 3 (heteroskedasticity) is a byproduct, and can be patched with robust SEs

$\implies$ We need a **curve**, not a line: something that starts near 0, rises steeply through the middle, and flattens near 1.

Outline

LPM vs. Logit vs. Probit



The LPM (dashed) overshoots at both ends. Logit and probit replace the line with an S-shaped curve

Where Does the S-Curve Come From?

Imagine each applicant has a **latent** (unobserved) "admissibility" score:

$$y_i^* = \beta_0 + \beta_1 \, \text{GPA}_i + \varepsilon_i$$

Latent just means we never see y_i^* directly. We only observe the binary decision:

$$\text{Admit}_i = \begin{cases} 1 & \text{if } y_i^* > 0 \\ 0 & \text{if } y_i^* \leq 0 \end{cases}$$

What is the probability of admission?

$$\begin{aligned} P(\text{Admit} = 1 \mid \text{GPA}) &= P(y^* > 0) \\ &= P(\varepsilon > -\beta_0 - \beta_1 \, \text{GPA}) \end{aligned}$$

$\Longrightarrow$ The distribution we assume for ε determines the shape of the curve.

From Latent Variable to Probability

Starting from:

$$P(\text{Admit} = 1) = P(\varepsilon > -\beta_0 - \beta_1 \, \text{GPA})$$

If ε has a *symmetric* distribution (by symmetry of the CDF):

$$P(\varepsilon > -z) = P(\varepsilon < z) = F(z)$$

So the probability is just the CDF of ε evaluated at $\beta_0 + \beta_1 \, \text{GPA}$:

$$P(\text{Admit} = 1 \mid \text{GPA}) = F(\beta_0 + \beta_1 \, \text{GPA})$$

$\Longrightarrow$ Any CDF maps $(-\infty, +\infty) \to [0, 1]$, which is exactly what we need. Two standard choices give us two models.

Two Distributions, Two Models

Logistic distribution for ε:

$$P(\text{Admit} = 1 \mid \text{GPA}) = \Lambda(\beta_0 + \beta_1\,\text{GPA}) = \frac{e^{\beta_0 + \beta_1\,\text{GPA}}}{1 + e^{\beta_0 + \beta_1\,\text{GPA}}}$$

This is the **logit** model.

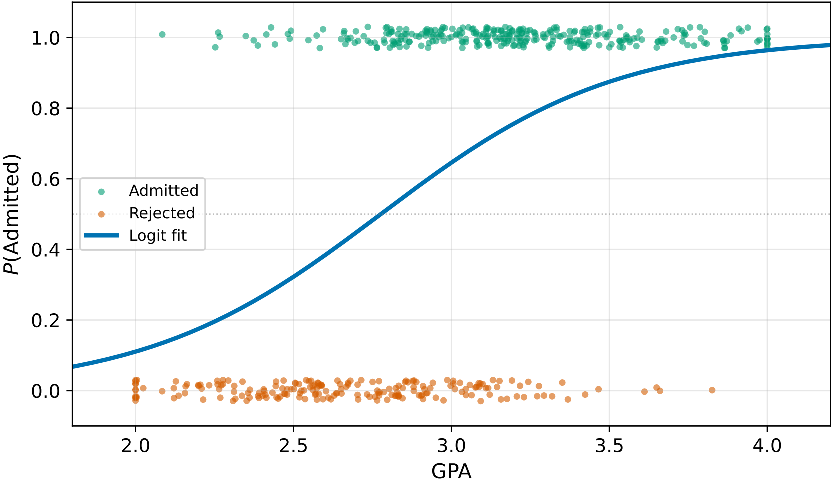
Standard normal distribution for ε:

$$P(\text{Admit} = 1 \mid \text{GPA}) = \Phi(\beta_0 + \beta_1\,\text{GPA})$$

This is the **probit** model.

Both produce S-shaped curves bounded in $[0, 1]$. The logistic CDF (Λ) has slightly heavier tails than the normal CDF (Φ), but in practice the two are nearly indistinguishable.

The Logit Model: Fitted Curve



The logit curve passes through the middle of the data, stays in $[0, 1]$, and has the steepest slope near $P = 0.5$.

Outline

The Interpretation Problem

Our logit estimates: $\hat{\beta}_0 = -7.46$, $\hat{\beta}_1 = 2.69$.

How would you interpret $\hat{\beta}_1 = 2.69$?

Tempting but wrong: "A one-unit increase in GPA raises the probability of admission by 2.69."

Why wrong? Because the logit is **nonlinear**. The coefficient 2.69 operates on the **log-odds** scale, not the probability scale. A probability change of 2.69 is not even possible.

$\Longrightarrow$ To interpret logit coefficients, we need to understand what they actually measure.

Log-Odds: What the Coefficient Measures

Define the **odds** of admission:

$$\text{Odds} = \frac{P(\text{Admit} = 1)}{P(\text{Admit} = 0)} = \frac{P}{1 - P}$$

The logit model is **linear in log-odds**:

$$\underbrace{\ln\left(\frac{P}{1 - P}\right)}_{\text{log-odds}} = \beta_0 + \beta_1 \,\text{GPA}$$

$\implies \beta_1 = 2.69$ means: a one-unit increase in GPA raises the **log-odds** of admission by 2.69.

Equivalently, the **odds ratio**:

$$e^{\beta_1} = e^{2.69} \approx 14.7$$

A one-unit increase in GPA **multiplies** the odds of admission by ≈ 14.7. For example, going from GPA 2.5 to 3.5 multiplies the odds by this factor.

Marginal Effects: What We Actually Want

The effect on *probability* depends on where you start:

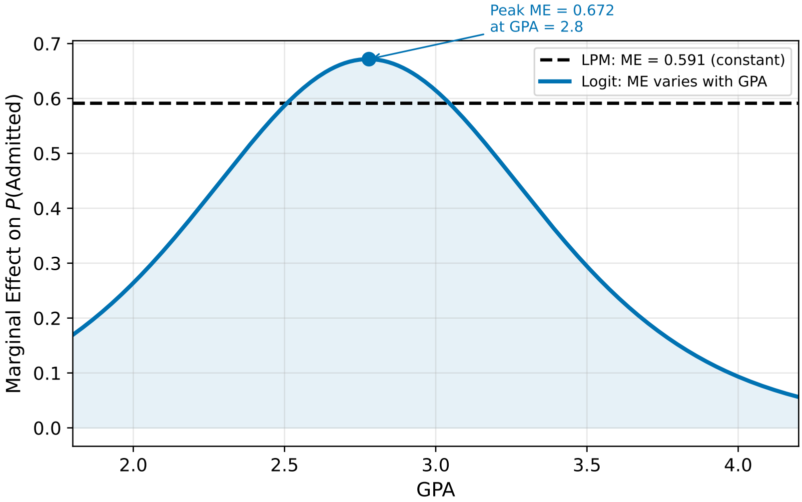
$$\underbrace{\frac{\partial P}{\partial\,\text{GPA}}}_{\text{marginal effect}} = \beta_1 \cdot \Lambda(\beta_0 + \beta_1\,\text{GPA}) \cdot \big(1 - \Lambda(\beta_0 + \beta_1\,\text{GPA})\big)$$

This equals $\beta_1 \cdot P \cdot (1 - P)$, which is largest when $P = 0.5$.

GPA	$\hat{P}$(Admit)	**Marginal Effect**
2.0	0.11	0.26
2.5	0.32	0.59
3.0	0.65	**0.61**
3.5	0.87	0.29
4.0	0.96	0.09

$\Longrightarrow$ The same one-unit GPA increase has roughly 7x more impact near the middle than at the top.

Marginal Effects: Visualized



The LPM assumes a constant effect (dashed). The logit captures the realistic bell shape: largest effect

Average Marginal Effect (AME)

Reporting a marginal effect at a single GPA is incomplete. Researchers typically report the **Average Marginal Effect**:

$$\text{AME} = \frac{1}{n}\sum_{i=1}^{n} \hat{\beta}_1 \cdot \hat{P}_i \cdot (1 - \hat{P}_i)$$

Compute the marginal effect *at each observation's actual GPA*, then average.

In our data: AME ≈ 0.49.

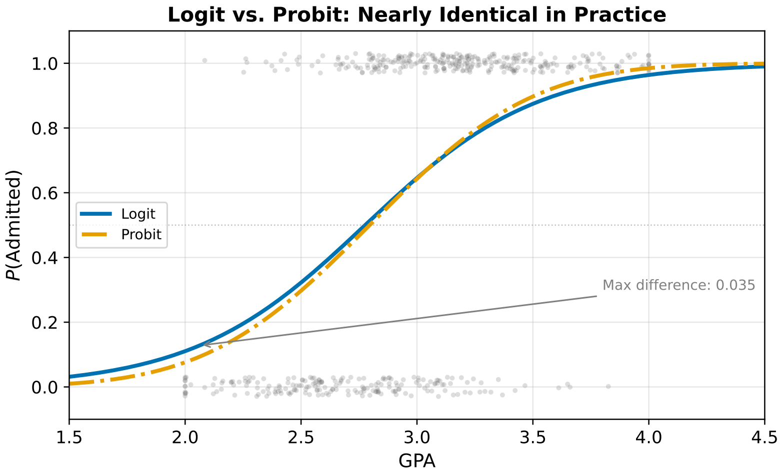
"On average, a one-unit increase in GPA is associated with a 49 percentage point increase in the probability of admission."

A full GPA point is a large change (e.g., 2.5 to 3.5), so this large AME makes sense in context.

$\implies$ AME gives a single summary number comparable to the LPM coefficient (0.59). The LPM overstates the average effect because it ignores diminishing returns.

Outline

Logit vs. Probit: Nearly Identical



The two curves are almost indistinguishable. The largest difference is in the tails, where both curves are near 0 or 1.

Logit vs. Probit: Coefficients

The logit and probit coefficients are on different scales:

	$\hat{\beta}_0$	$\hat{\beta}_1$	**Scale**
Logit	-7.46	2.69	Log-odds
Probit	-5.03	1.80	z-score (std. normal)

Three numbers you may see for the logit/probit coefficient ratio:

- $\sqrt{\pi^2/3} \approx 1.81$: the *theoretical* ratio, from the fact that the logistic distribution has variance $\pi^2/3$ while the standard normal has variance 1
- ≈ 1.6: a coarser textbook approximation (Amemiya)
- Here: $2.69/1.80 = 1.49$: the actual ratio in this finite sample

$\Longrightarrow$ Marginal effects and predicted probabilities are nearly identical regardless. The choice between logit and probit rarely changes conclusions. Logit is more common in economics because of the odds-ratio interpretation.

Outline

In Defense of the LPM

Despite its problems, the LPM is widely used in applied research. When is it acceptable?

The LPM works well when:

1. Predicted probabilities fall in $[0.2, 0.8]$ for most observations

   $\Longrightarrow$ The S-curve is approximately linear in this range

2. You only need the **average** effect, not predictions at extremes

   $\Longrightarrow$ LPM coefficient $\approx$ AME from logit

3. With robust standard errors to correct heteroskedasticity

The LPM fails when:

1. You need predictions (e.g., credit scoring, medical diagnosis)
2. The outcome is rare or very common (P near 0 or 1)
3. You have covariates that push predictions far from 0.5

LPM vs. Logit: Decision Framework

	LPM	**Logit / Probit**
Estimation	OLS	MLE
Predictions in $[0, 1]$?	No	Yes
Marginal effects	Constant	Vary with x
Coefficient $=$ ME?	Yes	No (need AME)
Heteroskedasticity	Built in	Handled by MLE
Speed / simplicity	Fastest	Slightly more complex
With FE (many groups)	Easy	Bias risk (incidental parameters problem)

(Incidental parameters: with many fixed effects, logit MLE estimates a parameter per group, which biases coefficients in short panels.)

→ LPM with robust SEs is a reasonable starting point. Switch to logit/probit when predictions tail

Outline

Why Not OLS?

OLS minimizes the sum of squared residuals. With binary y, this creates the problems we saw: impossible predictions, heteroskedasticity.

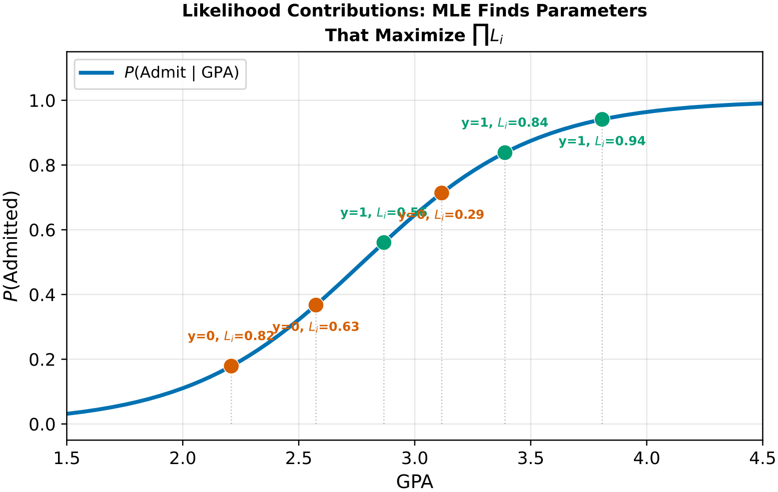
Logit and probit use a different estimation criterion: **Maximum Likelihood Estimation** (MLE).

MLE idea: Find the parameters β_0, β_1 that make the *observed data* most probable.

- For an admitted applicant ($y_i = 1$): we want P_i to be **high**
- For a rejected applicant ($y_i = 0$): we want P_i to be **low**

$\Longrightarrow$ MLE finds the S-curve that best separates the admitted from the rejected.

MLE: How It Works



Each observation contributes P_i (if admitted) or $1 - P_i$ (if rejected) to the likelihood. MLE maximizes the product of these contributions.

The Log-Likelihood

The **likelihood** for one observation:

$$L_i = P_i^{y_i} \cdot (1 - P_i)^{1-y_i}$$

The total **log-likelihood** (sum over all observations):

$$\ell(\beta_0, \beta_1) = \sum_{i=1}^{n} \left[y_i \ln P_i + (1 - y_i) \ln(1 - P_i) \right]$$

where $P_i = \Lambda(\beta_0 + \beta_1 \, \text{GPA}_i)$ for logit.

No closed-form solution $\implies$ solved numerically (Newton-Raphson, gradient ascent). Software handles this automatically.

$\implies$ MLE is the standard estimation method for logit and probit. The resulting $\hat{\beta}$ values are the ones that maximize this log-likelihood.

Thank you!

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