Chapter 10 Discussion Problems

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ECON 104 – UCLA

Outline

Problem 10.11: Wage Equation with IV

Consider the wage equation:

$$\log(WAGE) = \beta_1 + \beta_2 EXPER + \beta_3 EXPER^2 + \beta_4 EDUC + e$$

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Proposed instruments:

- $NEARC4 = 1$ if lived near 4-year college at age 10
- $NEARC2 = 1$ if lived near 2-year college at age 10

Problem 10.11 (a): Why These IV Might Work

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- Increase exposure to college-educated people
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Exogeneity: Family location when you're 10 years old:

- Determined by parents, not the child
- Should not directly affect adult earnings
- Only affects wages *through* education

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Step 1: Estimate first-stage regression:

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Save residuals $\hat{v}$.

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Step 3: Test $H_0 : \delta = 0$. If rejected $\Rightarrow$ endogeneity exists.

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Caution: Failing to reject doesn't *prove* exogeneity – we may just lack power.

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Interpretation: No evidence that the extra instrument is invalid.

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Consequences of weak IV:

- Standard errors inflate dramatically
- Confidence intervals become very wide
- 2SLS estimates may be unreliable

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$$\log(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EXPER + \beta_4 EXPER^2 + e$$

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Solution:
Binary indicators may be **more exogenous** because:

- Less correlated with unobserved family characteristics
- The "college vs no college" distinction captures a cleaner treatment effect

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F-test for $\gamma_2 = 0$: Check if $F > 10$.

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Result: Need to verify – if $F < 10$, the instrument is weak.

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Conclusion: Together, *MOTHERCOLL* and *FATHERCOLL* are **strong instruments**.

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Critical value: $\chi^2_{0.95,1} = 3.841$

Conclusion: $0.238 < 3.841 \Rightarrow$ **Fail to reject** validity.
The surplus instrument appears valid.

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Problem 10.20: CAPM with Measurement Error

The Capital Asset Pricing Model:

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f) + e$$

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Variables:

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- r_f = risk-free rate (30-day LIBOR)
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Problem: If market return is measured with error $\Rightarrow$ endogeneity.

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Interpretation:

- $\beta > 1 \Rightarrow$ **Riskier than market**
- When market goes up 1%, MSFT goes up $\approx 1.2\%$
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Confidence interval includes 1, but majority is above 1.

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- *RANK* preserves the ordering but removes measurement noise
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First-stage results:

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Conclusion: *RANK* is a **strong instrument**.

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95% CI: Entirely above 1 $\Rightarrow$ MSFT is definitively riskier than market.

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- $R^2 = 0.915$

Both instruments together are **very strong**.

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Conclusion: Fail to reject validity of surplus instrument.
Both instruments appear to be valid.

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Problem 10.23: Moment Conditions

Model: $\log(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EXPER + e$
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Solution:

$$\mathbb{E}[(y - \beta_1 - \beta_2 EDUC - \beta_3 EXPER) \cdot MOTHEREDUC] = 0$$
$$\mathbb{E}[(y - \beta_1 - \beta_2 EDUC - \beta_3 EXPER) \cdot EXPER] = 0$$
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$$\mathbb{E}[(y - \beta_1 - \beta_2 EDUC - \beta_3 EXPER)] = 0$$

Key insight: IV residuals are orthogonal to *instruments*, not to $EDUC$.

Problem 10.23 (b)–(c): Verifying Moment Conditions

- (b) Calculate $\sum \hat{e}_{IV}$, $\sum MOTHEREDUC \cdot \hat{e}_{IV}$, $\sum EXPER \cdot \hat{e}_{IV}$.

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All three sums ≈ 0 (up to numerical precision).

This confirms the moment conditions hold in the sample.

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Solution:

- $\sum EDUC \cdot \hat{e}_{IV} = 123.18 \neq 0$ (not orthogonal!)
- $\sum EDUC \cdot \hat{e}_{OLS} = 0$ (by construction)
- $\sum \hat{e}_{IV}^2 = 197.0 > \sum \hat{e}_{OLS}^2 = 190.2$

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All three sums ≈ 0 (up to numerical precision).
This confirms the moment conditions hold in the sample.

(c) Calculate $\sum EDUC \cdot \hat{e}_{IV}$ and $\sum \hat{e}_{IV}^2$. Compare to OLS.

Solution:

- $\sum EDUC \cdot \hat{e}_{IV} = 123.18 \neq 0$ (not orthogonal!)
- $\sum EDUC \cdot \hat{e}_{OLS} = 0$ (by construction)
- $\sum \hat{e}_{IV}^2 = 197.0 > \sum \hat{e}_{OLS}^2 = 190.2$

Key insight: IV does **not** minimize SSE – it solves moment conditions instead.

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Solution:

- $SSE_{IV} + SSR_{IV} \neq SST$ (decomposition fails!)
- $R_1^2 = SSR_{IV}/SST$ and $R_2^2 = 1 - SSE_{IV}/SST$ give different values

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OLS decomposition:

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Implication:

- R^2 has no clear interpretation for IV
- Cannot use R^2 to compare IV models
- Focus on coefficient estimates and their standard errors instead

Thank you!

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