The Hausman-Taylor Estimator

Estimating Time-Invariant Effects with Endogeneity

Jake Anderson

March 3, 2026

Outline

The Data

A policymaker wants to know the return to one more year of schooling. We have panel data on **200 workers** over **5 years**. How hard can it be?

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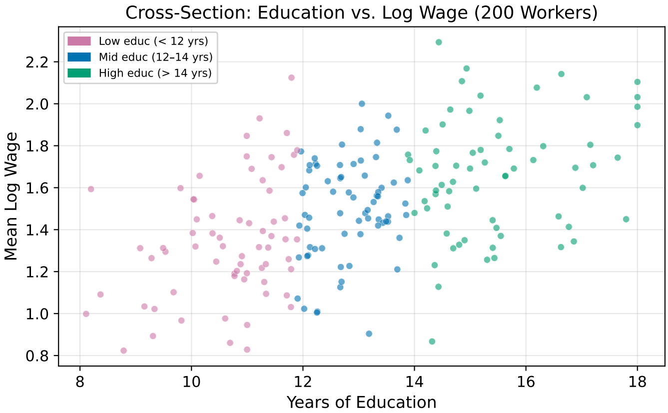

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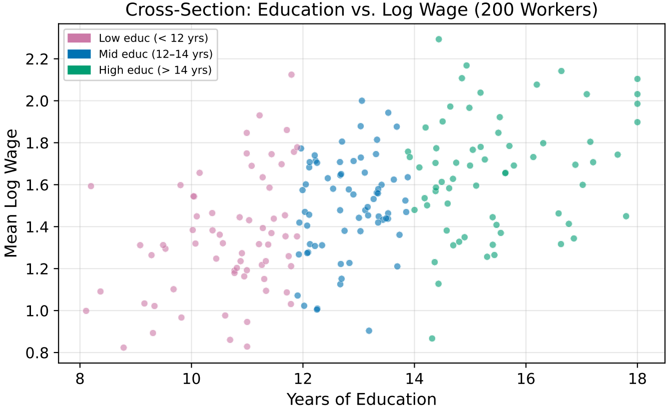

Higher education is associated with higher wages. But is the relationship **causal**?

Pooled OLS: A First Pass

Run pooled OLS of log wages on education, experience, union, and race:

$$\log(\text{wage}_{it}) = \beta_0 + \beta_1 \text{educ}_i + \beta_2 \text{exper}_{it} + \beta_3 \text{union}_{it} + \beta_4 \text{black}_i + \varepsilon_{it}$$

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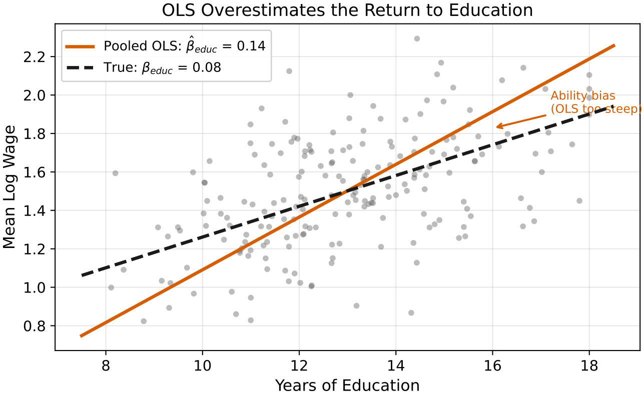

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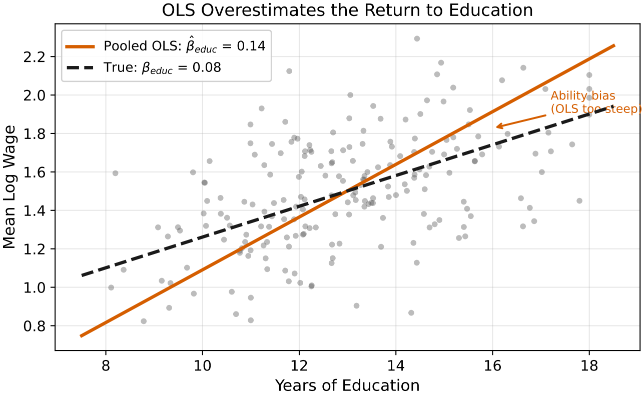

OLS estimates $\hat{\beta}_{\text{educ}} \approx 0.14$, but the true return is 0.08. Why the overestimate?

The Endogeneity Problem: Omitted Ability

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OVB formula (single-regressor version):

$$\hat{\beta}^{\text{OLS}}_{\text{educ}} = \underbrace{\beta_{\text{educ}}}_{\text{true}} + \underbrace{\beta_{\alpha}}_{\substack{\text{effect of ability} \\ \text{on wages}}} \times \underbrace{\delta_1}_{\substack{\text{relationship of} \\ \text{ability to educ}}}$$

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Both $\beta_\alpha > 0$ and $\delta_1 > 0$ $\implies$ OLS is **biased upward**.

Fixed Effects to the Rescue?

We know FE eliminates time-invariant unobservables. Write the model as:

$$\log(\text{wage}_{it}) = \alpha_i + \beta_1 \text{educ}_i + \beta_2 \text{exper}_{it} + \beta_3 \text{union}_{it} + \beta_4 \text{black}_i + \varepsilon_{it}$$

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$$\log(\text{wage}_{it}) - \overline{\log(\text{wage})}_i = \beta_2(\text{exper}_{it} - \text{ex\bar{p}er}_i) + \beta_3(\text{union}_{it} - \text{un\bar{i}on}_i) + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

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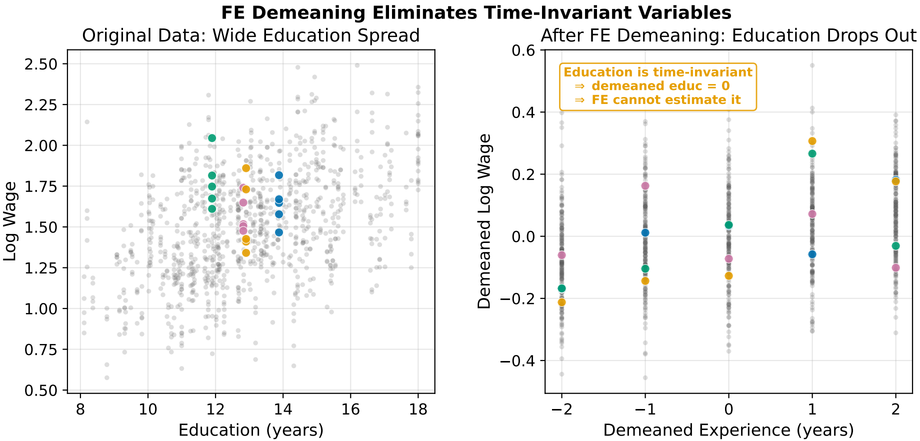
$\implies$ **Education is time-invariant.** Demeaning removes it entirely.

FE Demeaning: Education Disappears



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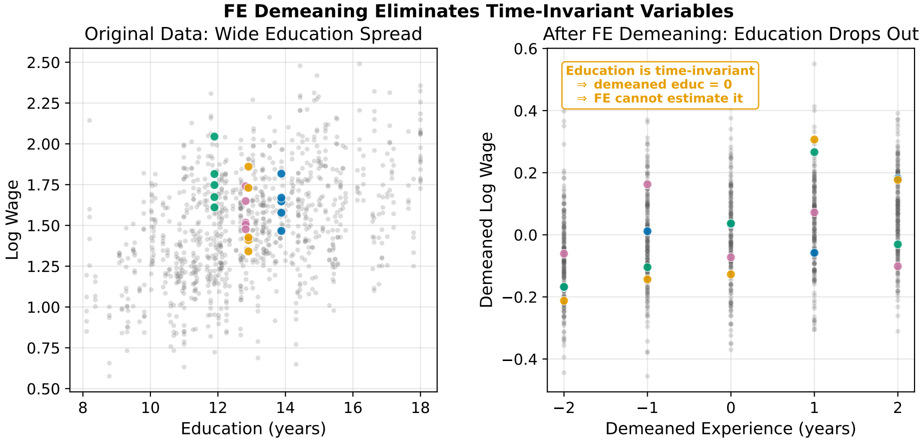

Each worker has **one education level** across all 5 years. After demeaning, the within-worker variation in education is exactly zero. FE cannot estimate β_{educ}.

Random Effects: Can It Estimate Education?

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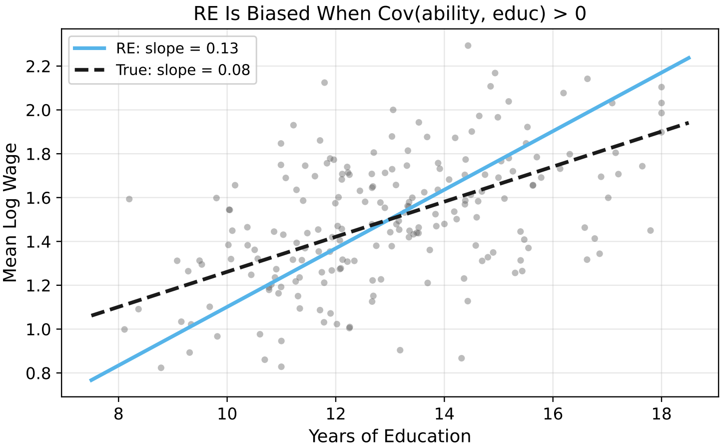
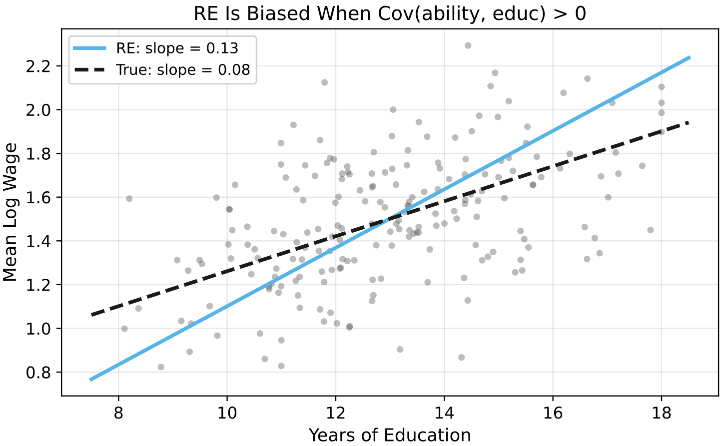
We just argued this is **violated** (ability correlates with education). So the RE estimate is **biased**.

RE Is Biased


RE Is Biased When Cov(ability, educ) > 0
RE: slope = 0.13
True: slope = 0.08
Mean Log Wage
Years of Education
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
8
10
12
14
16
18

RE Is Biased



RE estimates $\hat{\beta}_{\text{educ}} \approx 0.13$ (true $= 0.08$). The ability bias persists because RE does not fully eliminate α_i from the estimation.

The Dilemma

Abbreviations: TV = time-varying; TI = time-invariant.

Method	**Estimate $\hat{\beta}_{\textbf{educ}}$**	**Consistent?**	**Problem**
OLS	≈ 0.14	No	Omits ability
FE	N/A		Cannot estimate TI vars
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$\Longrightarrow$ We need an estimator that handles **time-invariant endogenous** variables.

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$\implies$ We already have instruments sitting inside the panel. We just need a procedure that uses them.

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The insight:

- Some time-varying variables (like experience) are **exogenous**: $\text{Cov}(\alpha_i, \text{exper}_{it}) = 0$
- Their **within-group means** ($\overline{\text{exper}}_i$) vary across individuals
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$\implies$ The within-group means of time-varying exogenous variables are **valid instruments** for education.

Variable Classification: Four Categories

HT requires classifying every variable into one of four groups:

Hausman-Taylor Variable Classification

	Exogenous	**Endogenous**
Time-Varying	$X_{1,it}$ Union (union_{it}) Experience (exper_{it})	$X_{2,it}$ (none in this example)
Time-Invariant	$Z_{1,i}$ Race (black_i)	$Z_{2,i}$ Corr(ability, educ) > 0 Education (educ_i)

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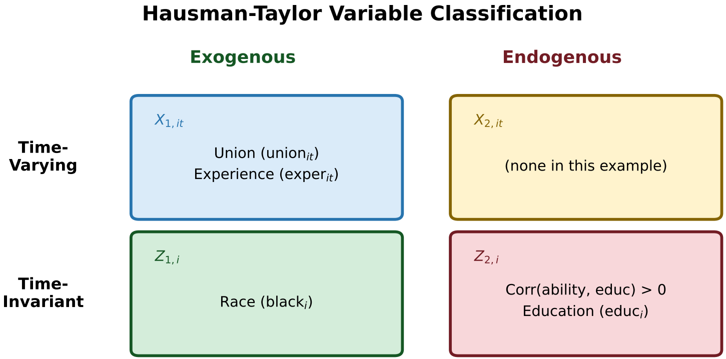
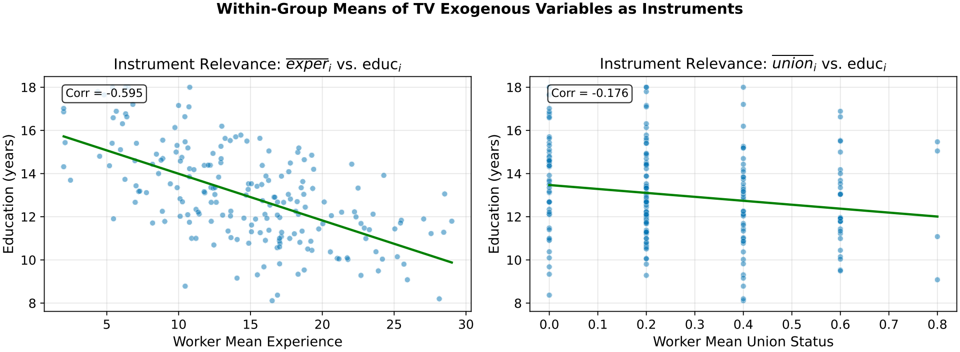


The classification determines which variables need instruments and which variables *provide* instruments.

The Instruments: Within-Group Means

Why do $\overline{\text{exper}}_i$ and $\overline{\text{union}}_i$ work as instruments for educ_i?



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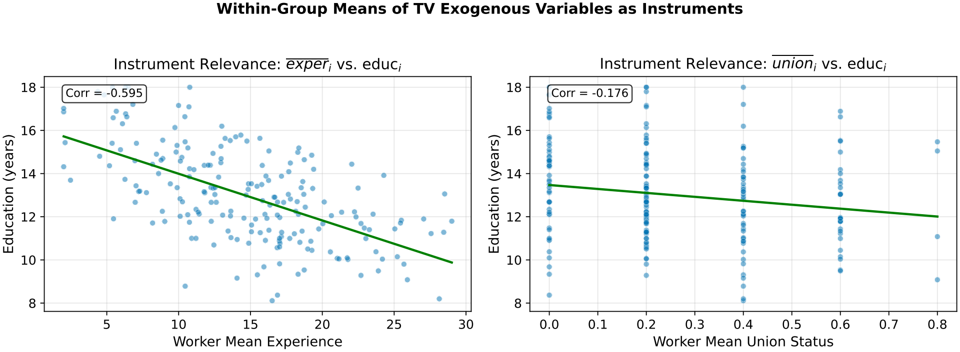

Relevance: Workers with more education have less experience (delayed entry) and lower unionization.
Validity: These means are uncorrelated with unobserved ability (exogenous by classification).

Instrument Requirements

Recall: X_1 = TV exogenous (exper, union); Z_2 = TI endogenous (educ).

For $\bar{X}_{1,i}$ (worker-level means of TV exogenous vars) to be valid instruments for $Z_{2,i}$:

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This is an **assumption**, not something we can verify directly. It follows from classifying $X_{1,it}$ as exogenous: if $\text{Cov}(\alpha_i, \text{exper}_{it}) = 0$ for all t, then $\text{Cov}(\alpha_i, \overline{\text{exper}}_i) = 0$.

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$\Longrightarrow$ Same logic as any IV estimation, but the instruments come from **within the panel**.

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$\implies$ We now have consistent $\hat{\beta}^{FE}_{\text{exper}}$ and $\hat{\beta}^{FE}_{\text{union}}$. We still need $\hat{\beta}_{\text{educ}}$ and $\hat{\beta}_{\text{black}}$.

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$\implies$ OLS on this equation would still be biased. We need instruments.

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$\Longrightarrow$ 2SLS isolates the variation in education that is driven by experience and union patterns, not by ability.

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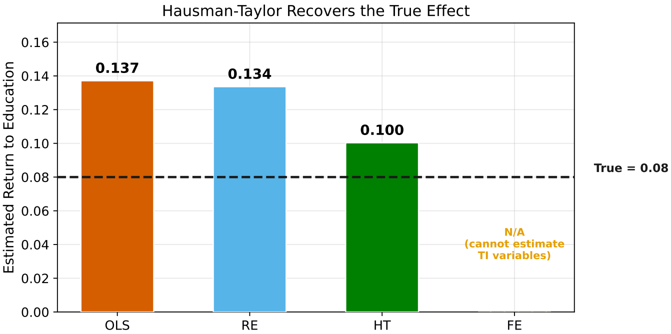
$\Longrightarrow$ You specify which variables are time-invariant and which are endogenous. The estimator does the rest.

The Result: HT Recovers the True Effect



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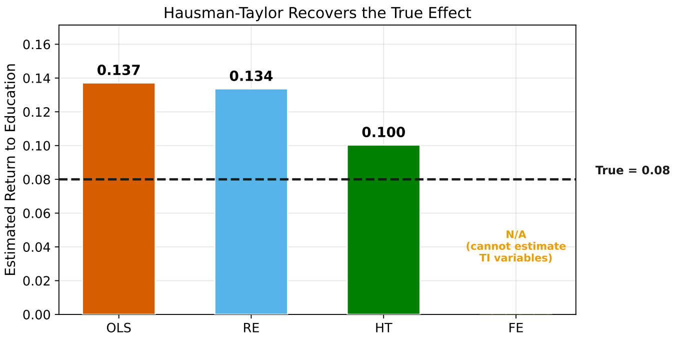

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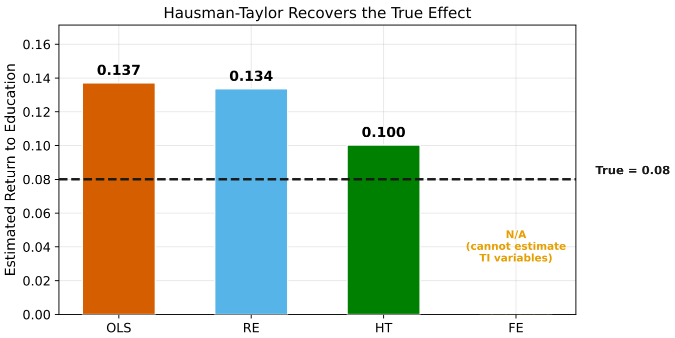

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$\implies$ By instrumenting education with within-group means of experience and union status, HT removes most of the ability bias.

Note: the HT estimate shown is from the IV-between step (Step 2b). The full FGLS estimator (Step 3) produces a slightly different number; Stata's `xthtaylor` implements the complete procedure.

How Each HT Component Solves a Problem

Problem	HT Component	Result
TV coefficients biased	FE (within estimation)	Consistent $\hat{\beta}_{\text{exper}}$, $\hat{\beta}_{\text{union}}$
TI endogenous variable	IV using $\bar{X}_{1,i}$ as instruments	Consistent $\hat{\beta}_{\text{educ}}$
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$\Longrightarrow$ HT fills the gap between FE (drops TI variables) and RE (requires full exogeneity).

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In our example: 2 TV exogenous (exper, union) instrumenting 1 TI endogenous (educ) $\implies$ overidentified (good: we can test validity).

Formal Assumptions

Partition the variables:

- $X_{1,it}$: time-varying, exogenous (exper, union)
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The Hausman-Taylor assumptions:

1. $\mathrm{E}[\varepsilon_{it} \mid X_1, X_2, Z_1, Z_2, \alpha_i] = 0$ (strict exogeneity of idiosyncratic error)

   The idiosyncratic error is pure noise.

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   Ability is unrelated to experience, union status, and race.

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$\Longrightarrow$ Assumption 2 is what makes $\bar{X}_{1,i}$ a valid instrument: it inherits exogeneity from $X_{1,it}$.

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$\Longrightarrow$ Rejection suggests that at least one of experience or union is not truly exogenous to ability. This would invalidate the HT approach.

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$\Longrightarrow$ This tests whether the TV variables you classified as exogenous truly are. It does *not* test the validity of the TI classification.

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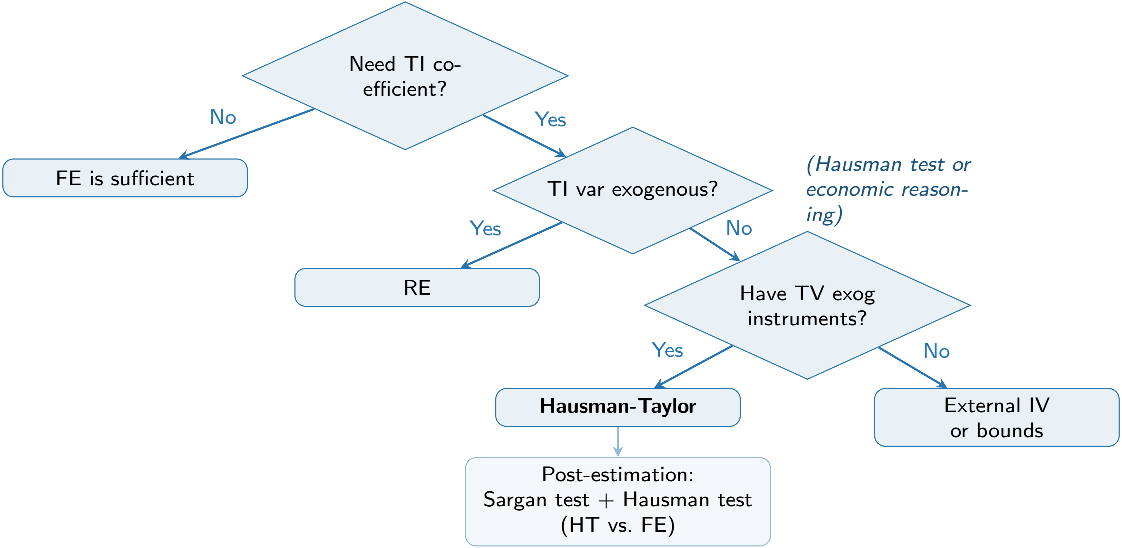
Decision Flowchart



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$\Longrightarrow$ When you need a time-invariant coefficient but suspect endogeneity, check whether your TV exogenous variables can serve as internal instruments.

Thank you!

jakeanderson@g.ucla.edu