Omitted Variable Bias

When Relevant Variables Are Left Out

Jake Anderson

March 3, 2026

The OLS Assumptions Under Random Sampling

Assumption	Description
RS1	The model is linear: $y_i = \beta_1 + \beta_2 x_i + e_i$
RS2	The data (y_i, x_i) are i.i.d.
RS3	**Exogeneity:** $E(e_i \mid x_i) = 0$
RS4	Homoskedasticity: $\text{Var}(e_i \mid x_i) = \sigma^2$
RS5	Rank condition: x_i takes at least two values
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Three sources of endogeneity: (1) omitted variable bias, (2) measurement error, (3) simultaneity.

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1. "The past is irrelevant!" $\implies Y = \beta_0 + \beta_1 X_1 + e_1$
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Question: Will $\hat{\beta}_1$ be the same in Model 1 and Model 3?

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$\implies$ OVB requires both: (1) omitted variable affects Y, and (2) omitted variable is correlated with included X.

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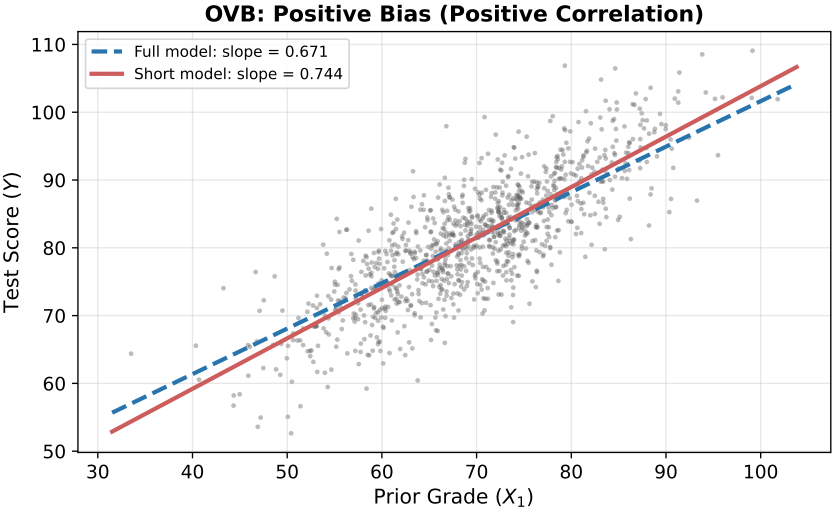
$\Longrightarrow$ The bias is $\hat{\beta}_2 \times \hat{\delta}_1$. If either is zero, there is no bias.

Visualizing OVB: Positive Bias



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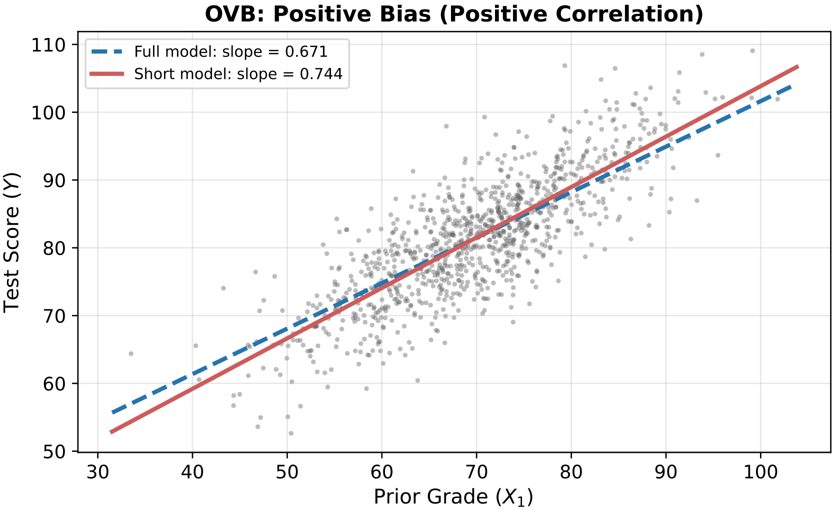

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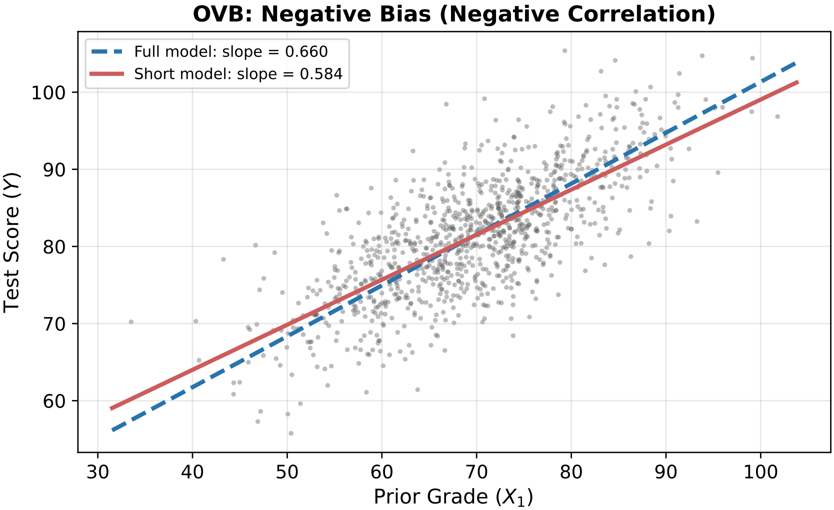

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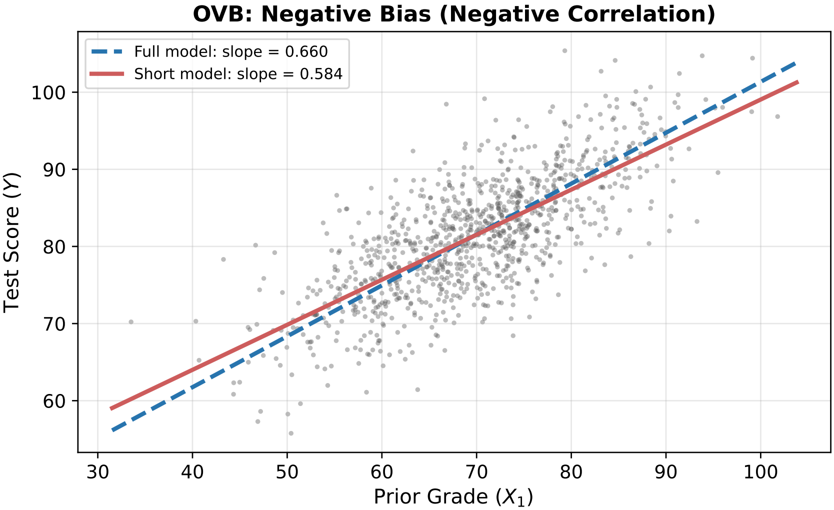

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Direction of Bias: The Sign Table

Direction of Omitted Variable Bias

β_{omitted} \ $\text{Corr}(X_{\text{included}}, X_{\text{omitted}})$	**Positive (+)**	**Negative (−)**
Positive (+)	**Positive bias (overestimate)**	**Negative bias (underestimate)**
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True DGP: $Y = 30 + 0.65X_1 + 0.40X_2 + \varepsilon$, with $\text{Corr}(X_1, X_2) \approx 0.4$

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$\hat{\beta}_1$ (Prior Grade)	≈ 0.73	—	≈ 0.65
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$\implies$ Both $\beta > 0$ and Corr > 0, so bias is positive in both short models.

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Solutions:

- Include the omitted variable (if observable)
- Use instrumental variables (if a valid instrument exists)
- Use fixed effects (if the omitted variable is time-invariant)

Thank you!

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