

Ordered Probit / Ordered Logit

Modeling Outcomes That Have a Ranking but Not a Scale

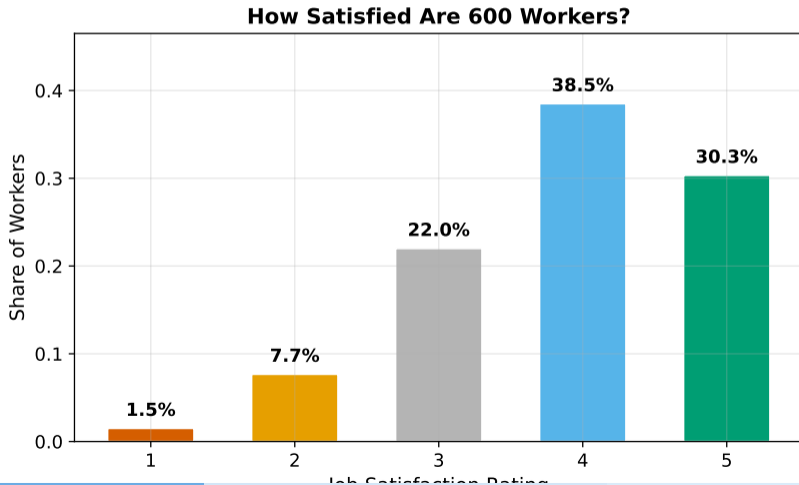
Jake Anderson

March 3, 2026

- 1 The Problem: OLS on Ordinal Outcomes
- 2 The Latent Variable Model
- 3 Interpretation and Marginal Effects
- 4 Ordered Choice vs. Multinomial Logit

The Data

A firm surveys **600 workers** about job satisfaction. Each worker reports a rating from 1 (very dissatisfied) to 5 (very satisfied).



Does Wage Predict Satisfaction?



Higher-paid workers tend to report higher ratings. But the outcome takes only five discrete values. How

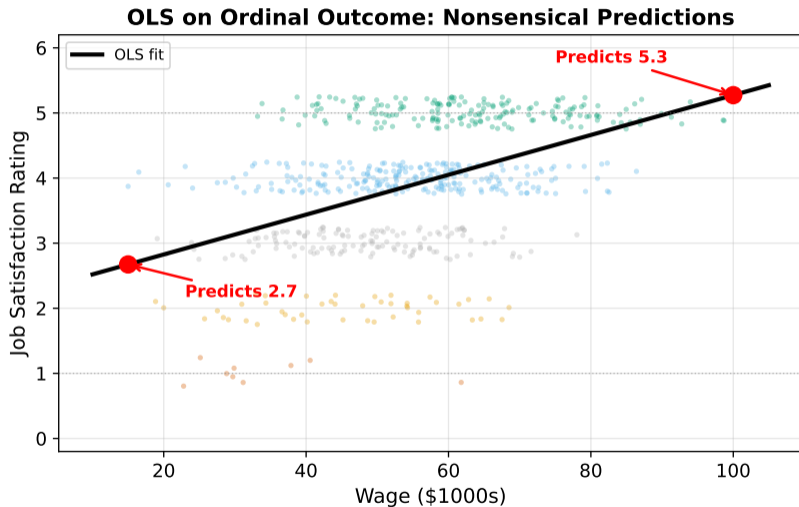
First Instinct: Run OLS

Treat the rating as a continuous number and regress it on wage:

$$\text{Rating}_i = \beta_0 + \beta_1 \text{Wage}_i + \varepsilon_i$$

This is fast and gives a slope you can interpret. What could go wrong?

OLS on Ordinal Outcomes: The Failure



At high wages, OLS predicts **5.3**. But the scale only goes to 5. At low wages, it predicts non-integer

What Goes Wrong with OLS on Ordinal Data

Three problems:

- 1 **Predictions outside the valid range.** OLS can predict 0.5 or 5.3 when the outcome only takes values 1–5
- 2 **Equal-spacing assumption.** OLS treats the jump from 1 to 2 as identical to the jump from 4 to 5. The numerical labels are arbitrary; replacing 1–5 with 2, 4, 6, 8, 10 would change the slope
- 3 **Constant marginal effects.** OLS forces each \$1k wage increase to add the same amount to the predicted rating, regardless of where on the scale the worker starts

⇒ We need a model that respects the ordinal nature of the outcome: categories have a ranking, but the distances between them are unknown.

What Would a Better Model Look Like?

A proper model for ordinal outcomes should:

- 1 **Produce valid probabilities.** $P(\text{Rating} = j) \in (0, 1)$ for each category, summing to 1
- 2 **Respect the ordering.** Use the fact that $5 > 4 > 3 > 2 > 1$, but not assume equal spacing
- 3 **Allow flexible marginal effects.** The effect of a wage increase should depend on where the worker currently sits on the satisfaction scale

⇒ Where can we find a model with these properties?

From Discrete Ratings to a Continuous Idea

Think about what the 1–5 scale really represents.

Satisfaction is probably **continuous** in a worker's mind. A worker who reports “4” and one who reports “5” might differ by a hair; another pair of 4 and 5 reporters might differ enormously.

The ratings are a **coarse discretization** of something continuous. What if we modeled that continuous variable directly and let the discrete ratings emerge from it?

⇒ This is the latent variable approach: posit an unobserved continuous satisfaction level, then map it to the observed categories through thresholds.

Outline

- 1 The Problem: OLS on Ordinal Outcomes
- 2 The Latent Variable Model**
- 3 Interpretation and Marginal Effects
- 4 Ordered Choice vs. Multinomial Logit

The Idea: A Continuous Variable Behind Discrete Ratings

Imagine each worker has a latent (unobserved) satisfaction level y_i^* that is continuous:

$$y_i^* = \beta_1 \text{Wage}_i + \beta_2 \text{Hours}_i + \beta_3 \text{Support}_i + \varepsilon_i$$

(no intercept; it is absorbed into the cutpoints, explained shortly)

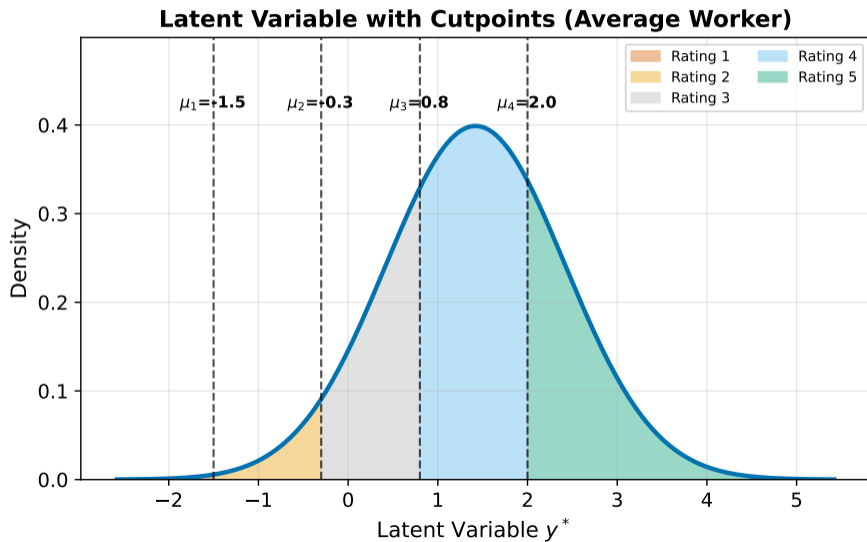
- y_i^* can take any real value (no boundary problems)
- We never observe y_i^* directly
- We observe only which **category** it falls into

The mapping from y_i^* to the observed rating uses **cutpoints** $\mu_1 < \mu_2 < \mu_3 < \mu_4$:

$$\text{Rating}_i = j \iff \mu_{j-1} < y_i^* \leq \mu_j$$

where $\mu_0 = -\infty$ and $\mu_5 = +\infty$.

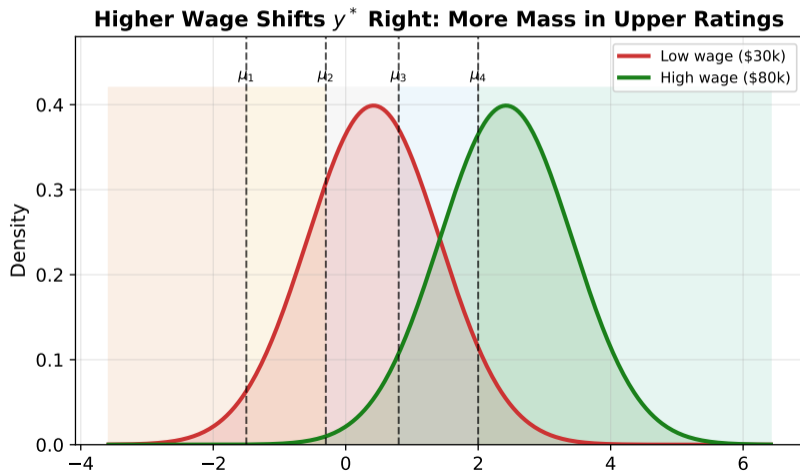
Cutpoints Partition the Latent Variable



How Covariates Shift the Distribution

Define the **linear index**: $XB_i = \beta_1 \text{Wage}_i + \beta_2 \text{Hours}_i + \beta_3 \text{Support}_i$.

A higher wage increases XB_i , which shifts the entire density of y_i^* to the right.



The Probability Formula: Derivation

The probability of observing rating j depends on where y_i^* falls:

$$P(\text{Rating}_i = j) = P(\mu_{j-1} < y_i^* \leq \mu_j)$$

Substitute $y_i^* = \mathbf{X}B_i + \varepsilon_i$ and rearrange:

$$= P(\mu_{j-1} - \mathbf{X}B_i < \varepsilon_i \leq \mu_j - \mathbf{X}B_i)$$

Since ε_i has CDF $F(\cdot)$:

$$= F(\mu_j - \mathbf{X}B_i) - F(\mu_{j-1} - \mathbf{X}B_i)$$

with boundary conditions $F(-\infty) = 0$ and $F(+\infty) = 1$.

Probit vs. Logit: Choosing $F(\cdot)$

The general formula works with any CDF. The two standard choices:

- **Ordered probit:** $F = \Phi$ (standard normal CDF), so $\varepsilon_i \sim N(0, 1)$
- **Ordered logit:** $F = \Lambda$ (logistic CDF), so $\varepsilon_i \sim \text{Logistic}$

\implies Same idea, different distributional assumption on ε_i . Results are usually similar in practice. Economists tend to use ordered probit; biostatisticians often prefer ordered logit.

What Gets Estimated

The model estimates two sets of parameters:

- 1 **Coefficients** $(\beta_1, \beta_2, \beta_3)$, i.e. $(\beta_{\text{wage}}, \beta_{\text{hours}}, \beta_{\text{support}})$

How each predictor shifts the latent variable y_i^*

- 2 **Cutpoints** $\mu_1, \mu_2, \mu_3, \mu_4$

Where the boundaries between adjacent categories lie

There is **no intercept** in the equation. An intercept would be absorbed into the cutpoints (you cannot separately identify both), so we normalize by omitting it.

The Parallel Regressions Assumption

The ordered model assumes that the coefficients $(\beta_1, \beta_2, \beta_3)$ are the **same for every cutpoint**. Visually: when a covariate changes, the density shifts horizontally, but all cutpoints stay fixed.

Where the name comes from: imagine running separate binary logits for each cumulative split:

- Rating ≤ 1 vs. > 1
- Rating ≤ 2 vs. > 2
- Rating ≤ 3 vs. > 3
- Rating ≤ 4 vs. > 4

The “parallel regressions” assumption says the slope on each predictor would be the **same across all four splits**. Only the intercept (cutpoint) differs.

⇒ This is what makes the ordered model parsimonious: one set of slopes instead of four.

Numeric Example: Computing Probabilities

Suppose a worker earns \$55k, works 42 hours, and has supervisor support (= 1). With the ordered probit coefficients $\beta_{\text{wage}} = 0.04$, $\beta_{\text{hours}} = -0.03$, $\beta_{\text{support}} = 0.80$:

$$XB_i = 0.04 \times 55 + (-0.03) \times 42 + 0.80 \times 1 = 2.20 - 1.26 + 0.80 = 1.74$$

With cutpoints $\mu_1 = -1.5$, $\mu_2 = -0.3$, $\mu_3 = 0.8$, $\mu_4 = 2.0$:

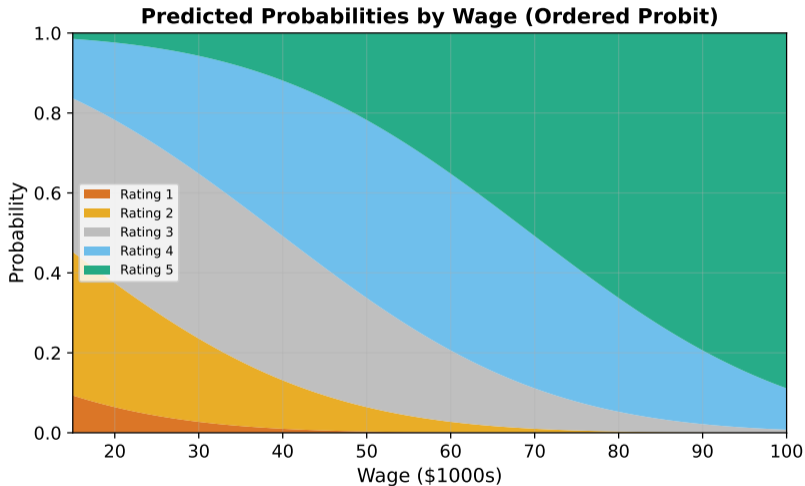
$$\begin{aligned} P(\text{Rating} = 4) &= \Phi(\mu_4 - 1.74) - \Phi(\mu_3 - 1.74) \\ &= \Phi(2.0 - 1.74) - \Phi(0.8 - 1.74) \\ &= \Phi(0.26) - \Phi(-0.94) \\ &= 0.603 - 0.174 = 0.429 \end{aligned}$$

\implies This worker has a 42.9% chance of reporting “Satisfied” (Rating 4). Software computes all five probabilities simultaneously.

Outline

- 1 The Problem: OLS on Ordinal Outcomes
- 2 The Latent Variable Model
- 3 Interpretation and Marginal Effects**
- 4 Ordered Choice vs. Multinomial Logit

Predicted Probabilities



As wage increases, probability shifts from lower ratings to higher ratings. At every wage level, the five probabilities sum to 1.

Coefficient Interpretation: Sign Only

The estimated coefficients tell us the **direction** in which a predictor shifts y_i^* :

Variable	$\hat{\beta}$	Interpretation
Wage (\$1k)	+	Higher wage $\implies y^*$ shifts right \implies higher satisfaction
Hours	-	More hours $\implies y^*$ shifts left \implies lower satisfaction
Supervisor support	+	Support $\implies y^*$ shifts right \implies higher satisfaction

\implies You can interpret the **sign**, but not the magnitude directly. Saying “a \$1k raise increases satisfaction by 0.04” is wrong because 0.04 is in latent-variable units, which have no natural scale.

For magnitudes, compute **marginal effects** on probabilities.

Marginal Effects: The Formula

The marginal effect on any single category depends on how much density sits near the two cutpoints that bracket that category.

The marginal effect of variable x_k on $P(\text{Rating} = j)$ is:

$$\frac{\partial P(\text{Rating} = j)}{\partial x_k} = \left[f(\mu_{j-1} - \mathbf{X}B_i) - f(\mu_j - \mathbf{X}B_i) \right] \cdot \beta_k$$

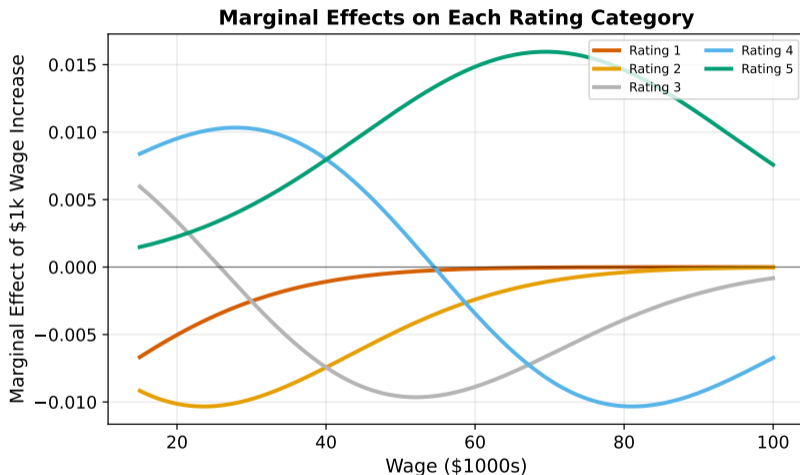
where $f(\cdot)$ is the density (derivative of F).

For the **extreme categories**, the boundary conditions simplify:

- $P(\text{Rating} = 1)$: Since $\mu_0 = -\infty$ and $f(-\infty) = 0$, the first term drops out.
ME = $-f(\mu_1 - \mathbf{X}B_i) \cdot \beta_k$. If $\beta_k > 0$, this is negative
- $P(\text{Rating} = 5)$: Since $\mu_5 = +\infty$ and $f(+\infty) = 0$, the second term drops out.
ME = $f(\mu_4 - \mathbf{X}B_i) \cdot \beta_k$. If $\beta_k > 0$, this is positive

⇒ A positive coefficient always decreases $P(\text{lowest})$ and increases $P(\text{highest})$. But what about the middle categories?

Middle Categories: Ambiguous Marginal Effects



The marginal effect on **middle categories changes sign** depending on where the worker starts. At low wages, a raise increases $P(\text{Rating} = 4)$. At high wages, it decreases it (because those workers are

Estimation: Maximum Likelihood

Each worker contributes one term to the likelihood: the probability of the rating they actually reported.

The log-likelihood:

$$\ell = \sum_{i=1}^N \sum_{j=1}^5 d_{ij} \ln P(\text{Rating}_i = j)$$

where $d_{ij} = 1$ if worker i reported rating j .

- $P(\text{Rating}_i = j) = F(\mu_j - \mathbf{X}B_i) - F(\mu_{j-1} - \mathbf{X}B_i)$
- Parameters: $(\beta_1, \beta_2, \beta_3)$ and μ_1, \dots, μ_4
- No closed-form solution \implies numerical optimization (same as binary logit)

\implies Software estimates the coefficients and the cutpoints jointly.

Outline

- 1 The Problem: OLS on Ordinal Outcomes
- 2 The Latent Variable Model
- 3 Interpretation and Marginal Effects
- 4 Ordered Choice vs. Multinomial Logit**

This Is Not Multinomial Logit

Ordered choice and multinomial logit both handle categorical outcomes. But they are built for different situations.

Multinomial logit (previous deck): categories have **no natural ordering**

- Car, Bus, Bike, Walk \implies no sense in which Car $>$ Bus
- Each alternative gets its **own coefficient** (β_j per mode)

Ordered choice (this deck): categories have a **natural ranking**

- Rating 5 $>$ 4 $>$ 3 $>$ 2 $>$ 1
- One set of coefficients (β_1, \dots, β_K) shifts the entire latent distribution
- The ordering provides information that the model exploits

\implies Using multinomial logit on ordered data wastes the ordering information and estimates far more parameters than necessary ($J - 1$ coefficient vectors instead of one).

When Parallel Regressions Fails

Recall the parallel regressions assumption: the same slopes apply at every cumulative split. When does this break down?

Example: suppose wage has a strong effect on moving from “Dissatisfied” to “Neutral,” but no effect on moving from “Satisfied” to “Very Satisfied.” The ordered model cannot capture this because it forces a single β_{wage} .

Test: the Brant test checks whether the slopes are stable across cutpoints (note: applies specifically to ordered logit). If it rejects, consider:

- A **generalized ordered logit** (which allows slopes to vary by cutpoint)
- Multinomial logit as a fallback

⇒ Always check parallel regressions before reporting ordered model results.

Ordered Probit vs. Ordered Logit

The only difference is the assumed distribution of ε_i :

	Ordered Probit	Ordered Logit
Error distribution	$\varepsilon_i \sim N(0, 1)$	$\varepsilon_i \sim \text{Logistic}$
CDF used	$\Phi(\cdot)$	$\Lambda(\cdot) = \frac{e^{(\cdot)}}{1+e^{(\cdot)}}$
Tails	Thinner	Slightly heavier
Predicted probabilities	Nearly identical	Nearly identical

⇒ In practice, both give very similar results. Choose based on convention in your field.

Decision Framework: Which Model to Use

① **Outcome is binary?** (yes/no, 0/1)

⇒ Binary logit or probit

② **Outcome is categorical with no natural order?** (car, bus, bike, walk)

⇒ Multinomial logit

③ **Outcome is categorical with a natural order?** (strongly disagree → strongly agree)

⇒ Ordered probit or ordered logit

④ **Parallel regressions assumption fails?**

⇒ Generalized ordered logit, or fall back to multinomial logit

⇒ The ordered model is more efficient than multinomial logit when the ordering is genuine, because it estimates fewer parameters while exploiting the ranking structure.

Summary: Back to Job Satisfaction

Recall the three OLS problems. How does ordered probit address each one?

① **Predictions outside the valid range**

⇒ Ordered probit produces probabilities $P(\text{Rating} = j) \in (0, 1)$ that sum to 1. No predictions of 5.3

② **Equal-spacing assumption**

⇒ The cutpoints μ_1, \dots, μ_4 are estimated freely. The model never assumes equal distances between categories

③ **Constant marginal effects**

⇒ Marginal effects depend on the worker's current position. A raise affects a dissatisfied worker differently than one who is already satisfied

⇒ Interpret the sign of $\hat{\beta}$ for direction; compute AMEs for magnitude. Check the parallel regressions assumption before reporting.

Thank you!
jakeanderson@g.ucla.edu