Ridge, Lasso, and Elastic Net

When More Features Means Worse Predictions

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Outline

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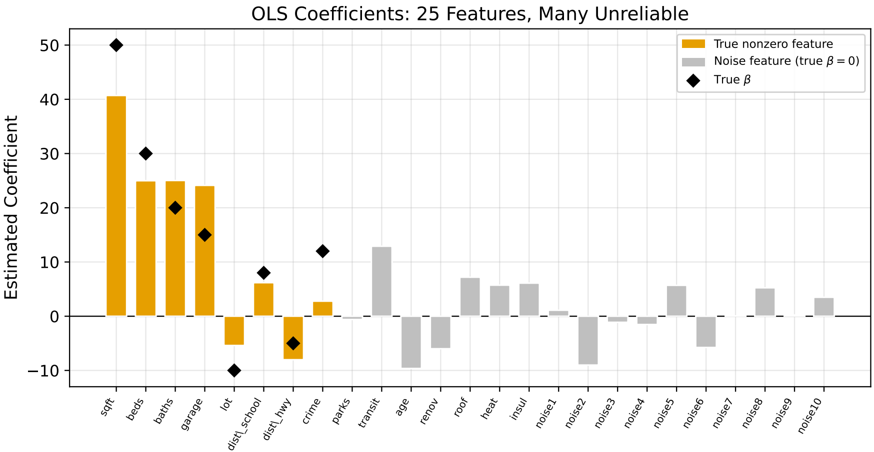
What went wrong?

OLS with 25 Features: Overfitting



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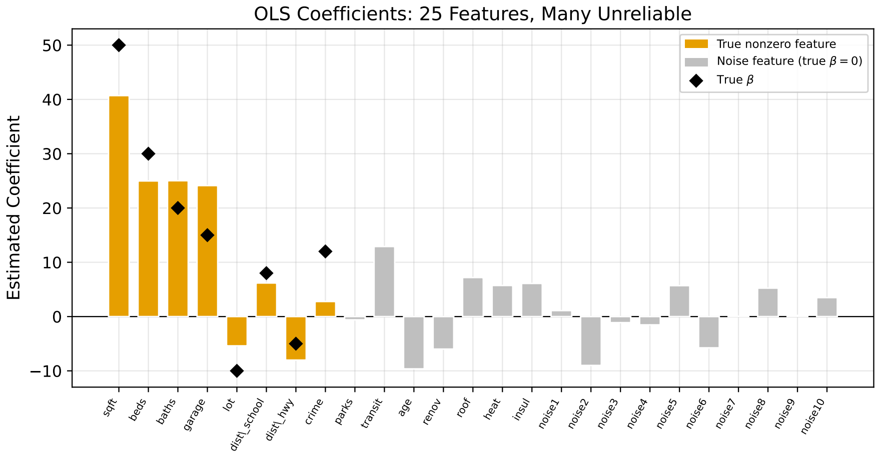

OLS assigns large coefficients to many features. Some of these estimated effects are real. Others are noise that OLS is fitting by mistake. But from the OLS output alone, you cannot tell which is which.

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$\Longrightarrow$ We need a method that deliberately sacrifices some in-sample fit to gain better out-of-sample prediction.

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$\Longrightarrow$ OLS failed because it treated noise features as if they were real, and multicollinearity made even the real estimates unreliable.

A Different Goal: Prediction, Not Causation

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$\Longrightarrow$ Regularization is a prediction tool. It deliberately introduces bias to reduce variance.

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What kind of penalty should we use? The three main choices: Ridge (ℓ_2), Lasso (ℓ_1), and Elastic Net (both). The subscript tells you the exponent: ℓ_2 squares each coefficient, ℓ_1 takes absolute values.

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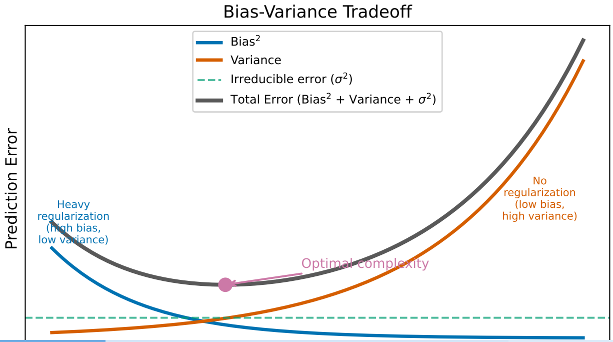

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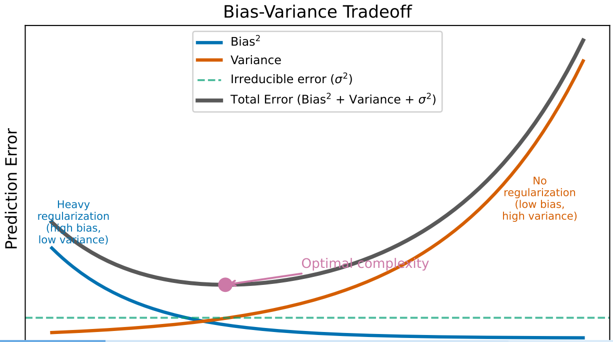

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Note: we standardize all features before fitting (subtract mean, divide by SD). This ensures the penalty treats all features equally, regardless of their original scale. The intercept is not penalized.

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$\Longrightarrow$ Ridge always keeps all features in the model. It reduces their influence but does not remove them.

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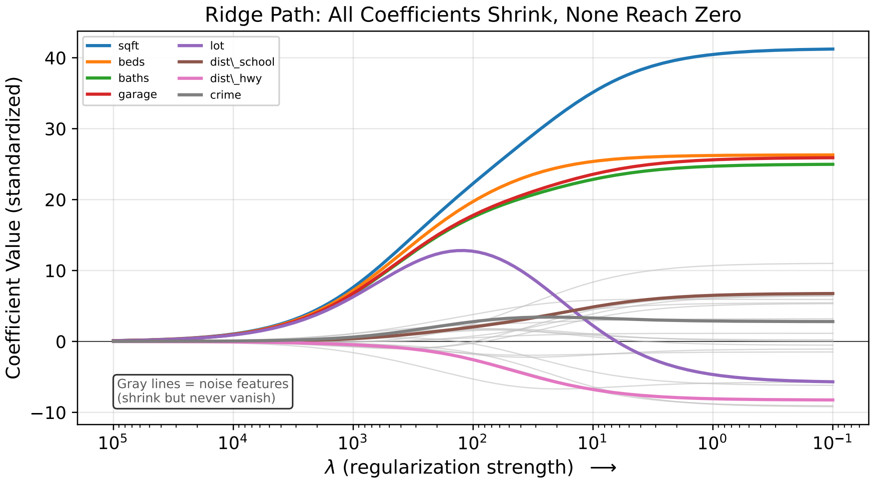

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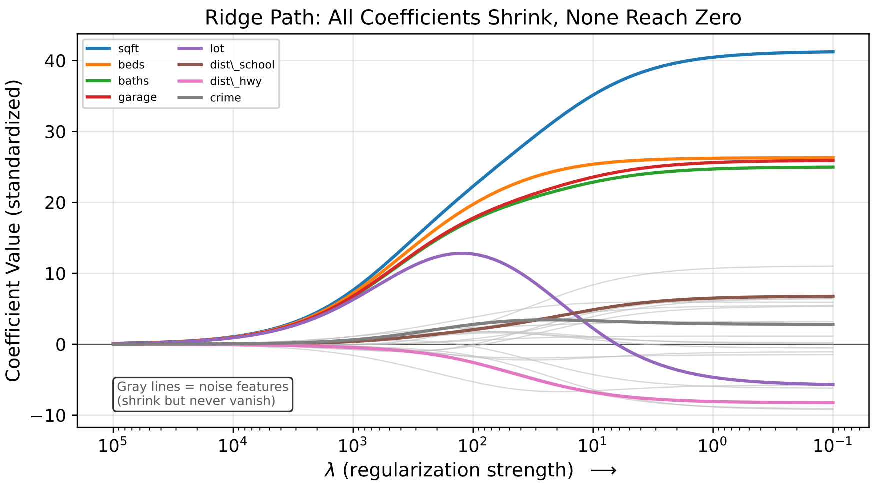

As λ increases (left to right), every coefficient shrinks toward zero. The noise features (gray) become negligible, but no coefficient ever reaches exactly zero.

Ridge Properties

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$\implies$ Ridge is a workhorse for prediction with correlated features, but it does not tell you which features to drop.

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All we need to change is the shape of the penalty.

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- The ℓ_2 penalty (Ridge) costs more as $|\beta_j|$ grows, so it pushes large coefficients hard but barely touches small ones.
- The ℓ_1 penalty (Lasso) charges a **constant rate** per unit of $|\beta_j|$, so it pushes small coefficients all the way to zero. If $\beta = 0.1$, the ℓ_2 cost is 0.01 (negligible) but the ℓ_1 cost is 0.1 (same rate as for $\beta = 10$).

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$\Longrightarrow$ Lasso performs **feature selection**: it sets some $\hat{\beta}_j$ to exactly zero.

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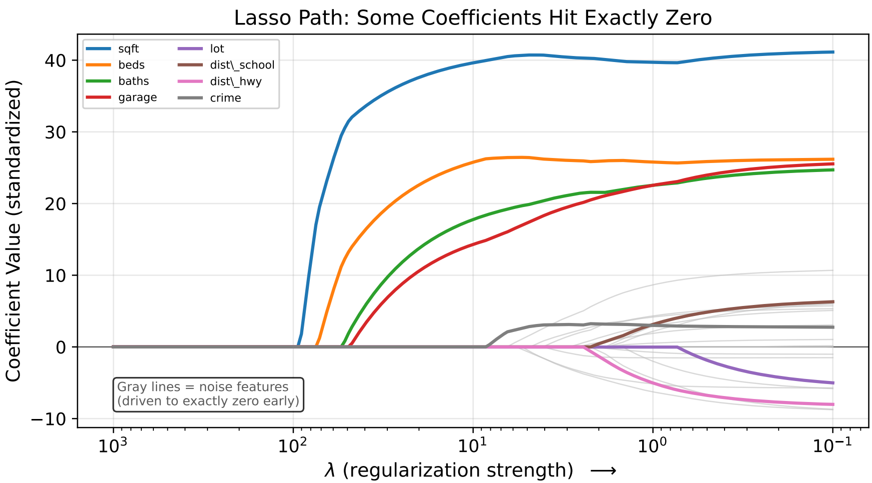

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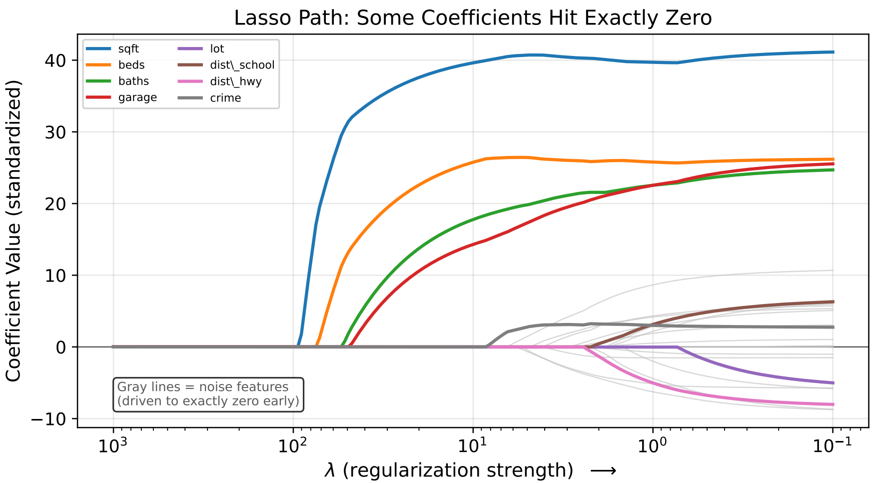
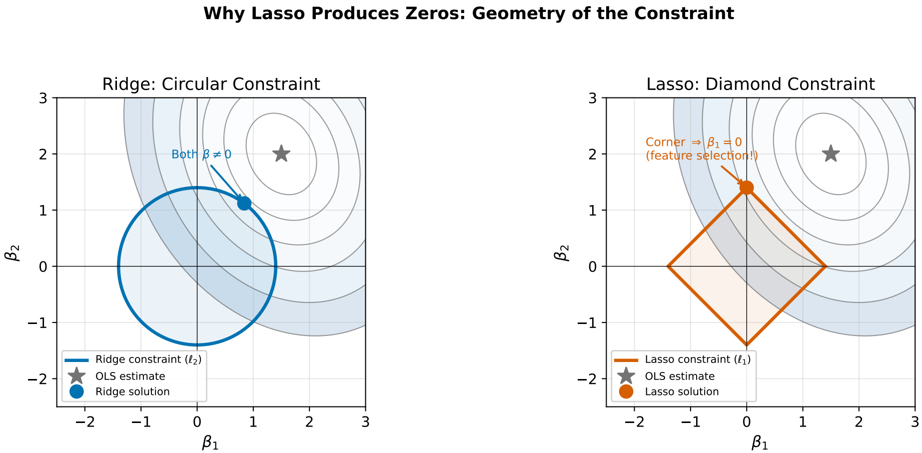


As λ increases, noise features (gray) are driven to zero **first**. The true signal features persist longer. At the right level of λ, only the genuinely predictive features remain.

Why Does Lasso Produce Zeros? The Geometry



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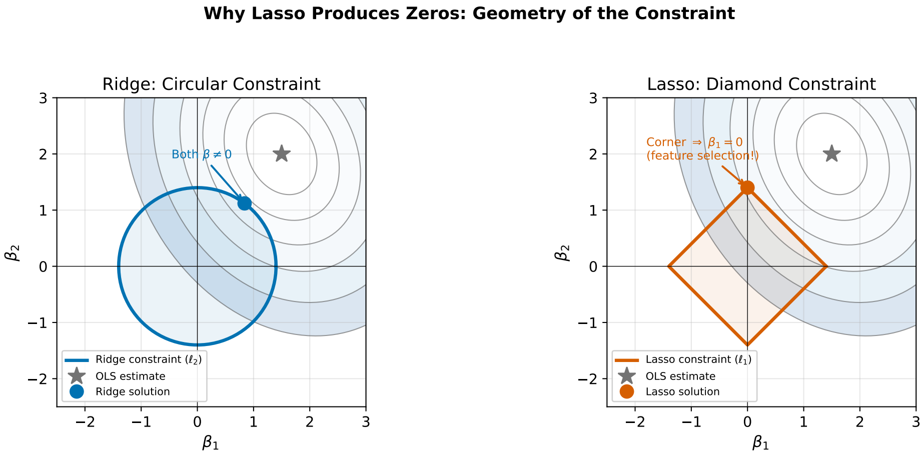

Each ellipse is a set of (β_1, β_2) pairs giving the same RSS. As we shrink the constraint region, the first contact point is the solution. Ridge uses a **circle** ($\beta_1^2 + \beta_2^2 \leq t$); Lasso uses a **diamond** ($|\beta_1| + |\beta_2| \leq t$).

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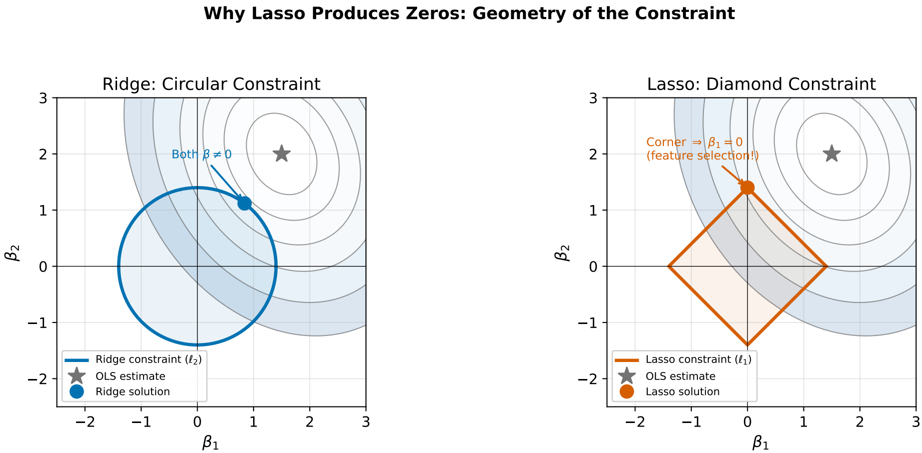

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$\Longrightarrow$ Lasso serves double duty: it improves prediction *and* identifies which variables to keep.

Lasso Properties

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$\Longrightarrow$ When features come in correlated groups (as in our house data), Lasso's selection can be unstable. This motivates combining the two penalties.

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$\Longrightarrow$ Combine the ℓ_1 and ℓ_2 penalties into a single objective.

Outline

Elastic Net: Combining ℓ_1 and ℓ_2

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Two tuning parameters:

- $\lambda \geq 0$: overall regularization strength (same role as before)
- $\alpha \in [0, 1]$: the **mixing parameter** between Lasso ($\alpha = 1$) and Ridge ($\alpha = 0$)

Note: this α is unrelated to the significance level from hypothesis testing. The $\frac{1}{2}$ in the ℓ_2 component is a standard convention; since λ is chosen by CV, this scaling is absorbed into tuning.

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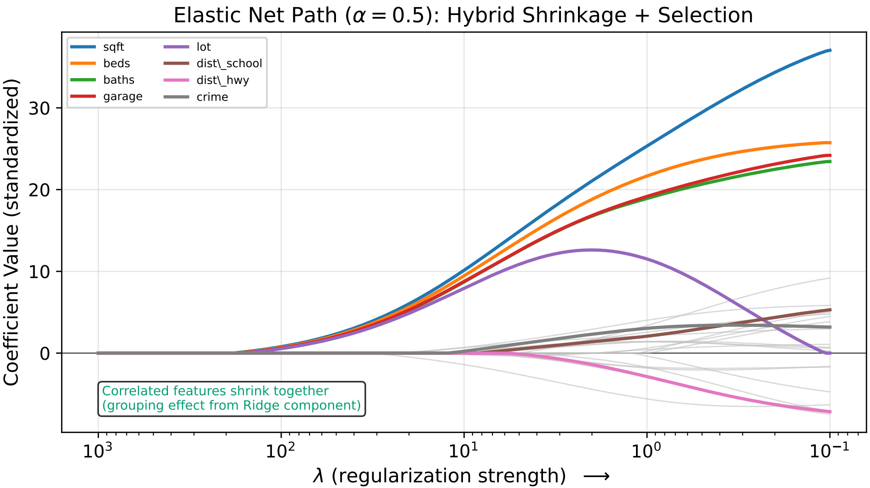
α	**Method**	**Behavior**
$\alpha = 1$	Lasso	Feature selection, no grouping
$\alpha = 0$	Ridge	Shrinkage only, no selection
$0 < \alpha < 1$	Elastic Net	Selection + grouping

Elastic Net Coefficient Path



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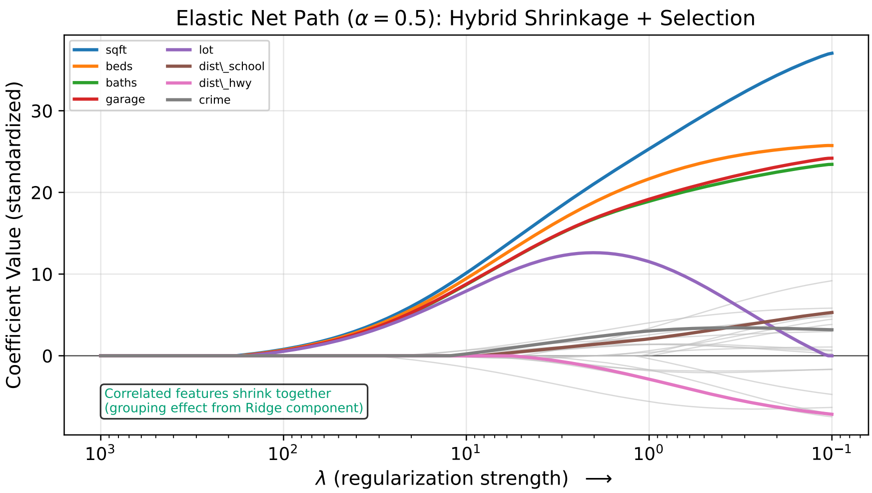

Like Lasso, noise features are driven to zero. Like Ridge, correlated features within the same block

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$\Longrightarrow$ Elastic Net inherits the best of both worlds: sparsity from Lasso and stability from Ridge.

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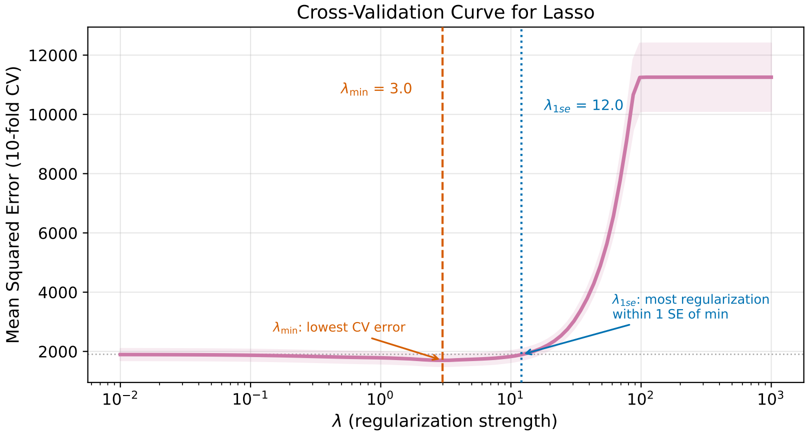
$\implies$ Cross-validation simulates out-of-sample prediction using only the training data. It selects λ without touching the test set.

The CV Curve: $\lambda_{\min}$ and λ_{1se}



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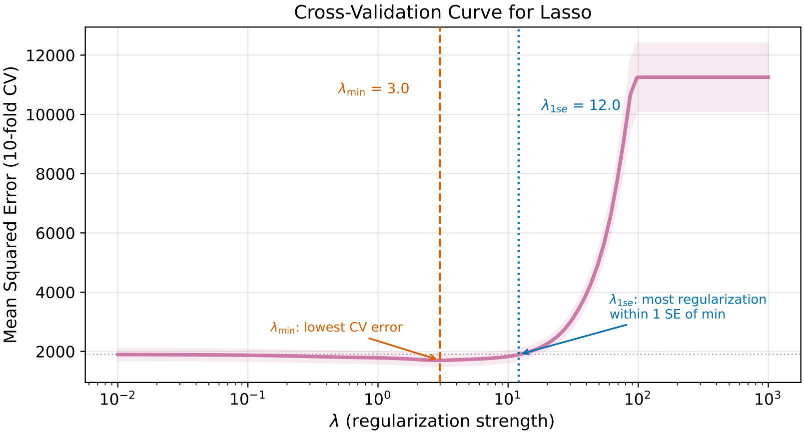

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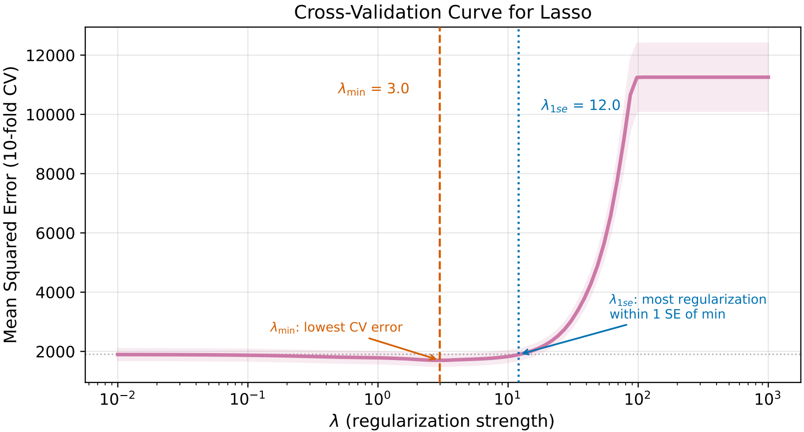

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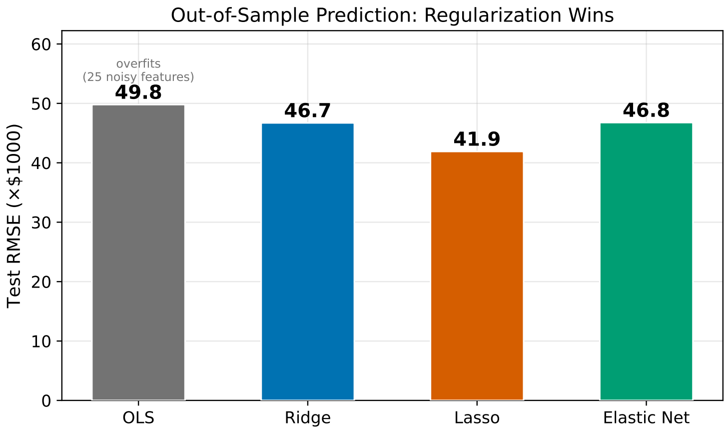

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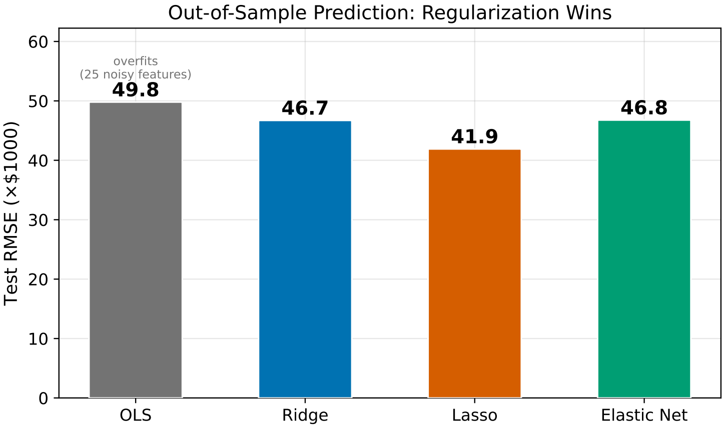

All three regularized methods beat OLS on out-of-sample prediction. Ridge, Lasso, and Elastic Net perform similarly here; the improvement comes from penalizing noise features.

Ridge vs. Lasso vs. Elastic Net: When to Use Which

Scenario	**Ridge**	**Lasso**	**EN**
Many correlated features	✓		✓
Want feature selection		✓	✓
Few true signals, many noise		✓	✓
$p > n$	✓	(limited)	✓
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Rules of thumb:

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$\implies$ In all cases, choose λ by cross-validation. Never choose λ by hand or by in-sample fit.

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Suppose your Lasso model drops “distance to school.” Does that mean distance has no effect on price?
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$\Longrightarrow$ Use regularization for prediction. Use IV, FE, or other causal methods for inference. Do not mix them up.

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$\Longrightarrow$ Regularization is a prediction tool: it trades bias for lower variance. For causal inference, use IV, FE, or RE instead.

Thank you!

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