# The Tobit Model (Censored Regression)

## When 40% of Your Data Is Piled Up at Zero

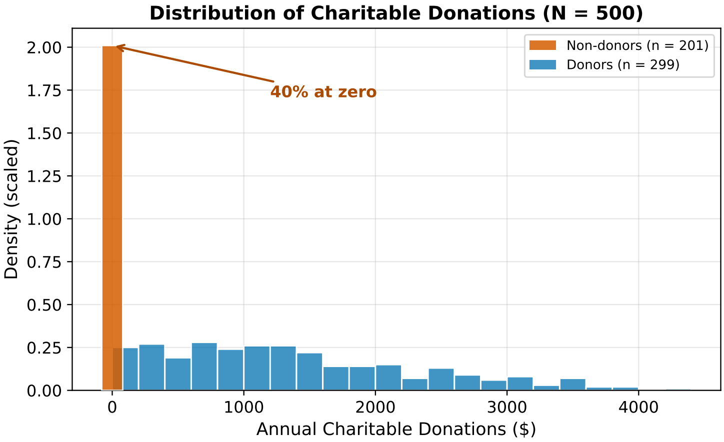
Jake Anderson

March 3, 2026

# Outline

# The Data: Charitable Donations

A researcher surveys **500 households** and records **annual charitable donations** (\$). Covariates include household income (\$1000s), years of education, and number of children.



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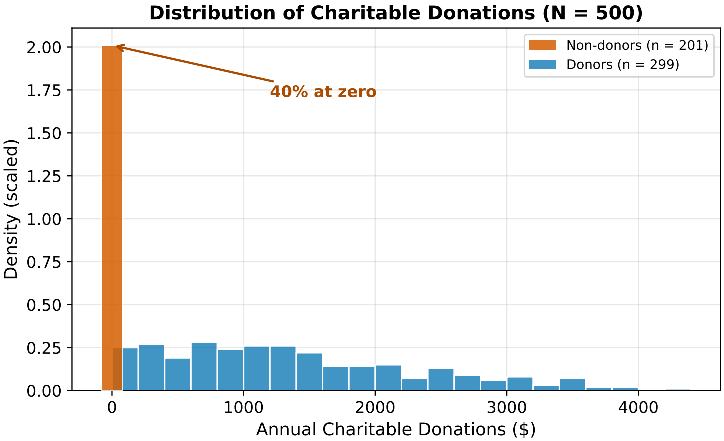


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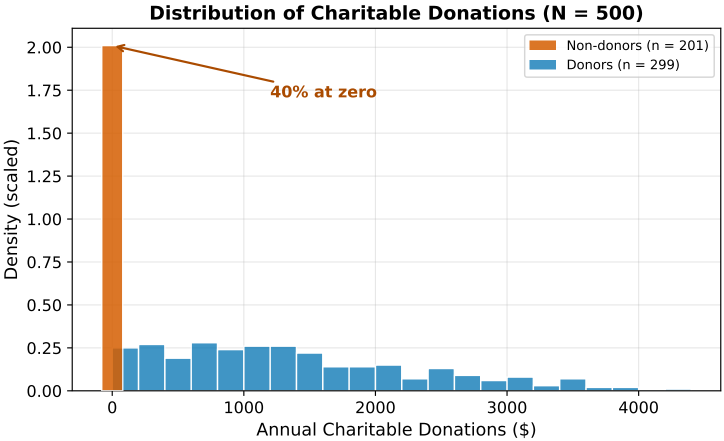


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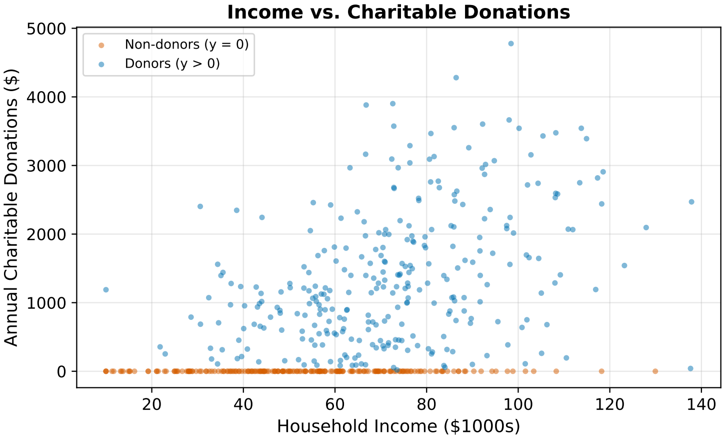

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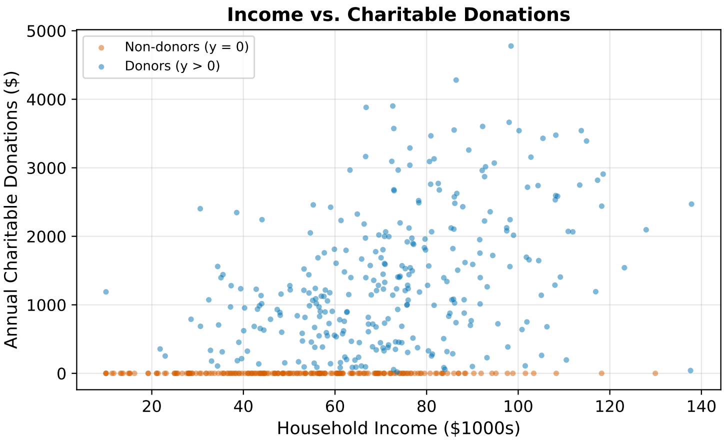


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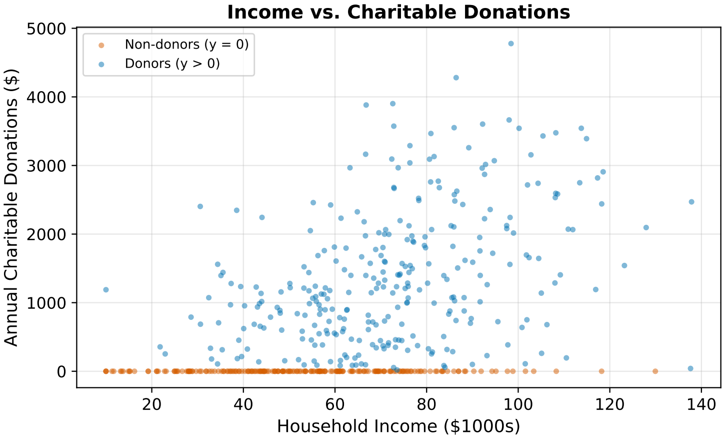


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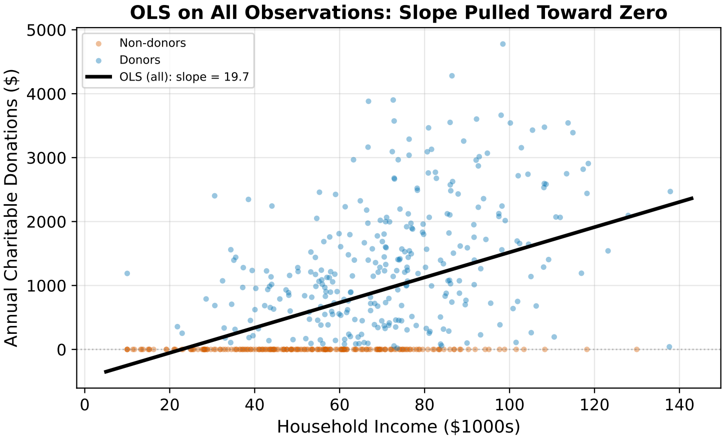

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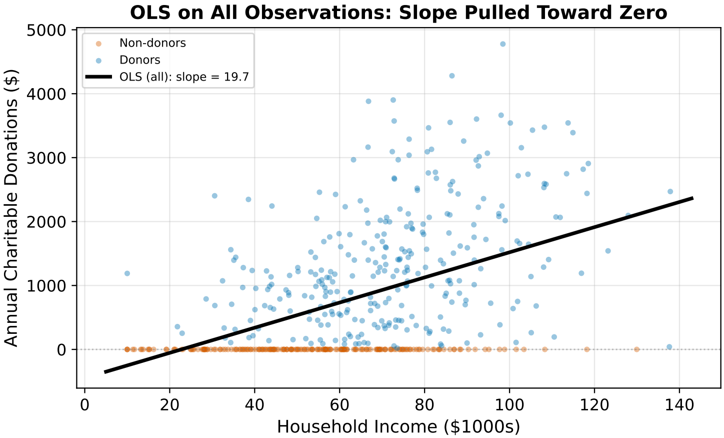


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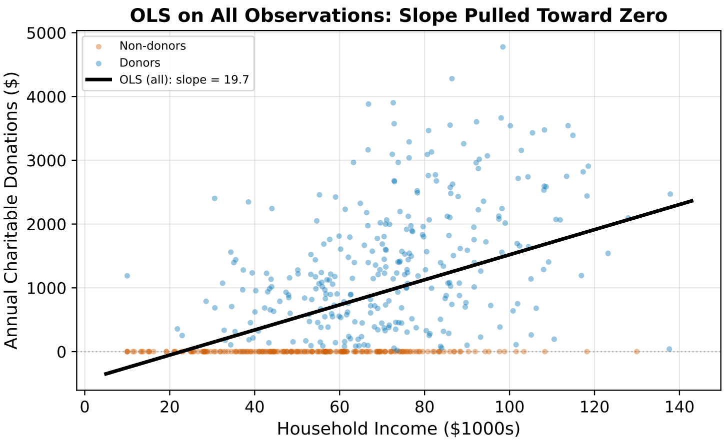


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Why is OLS attenuated? The 200 non-donors all sit at $y = 0$ regardless of their income. OLS treats

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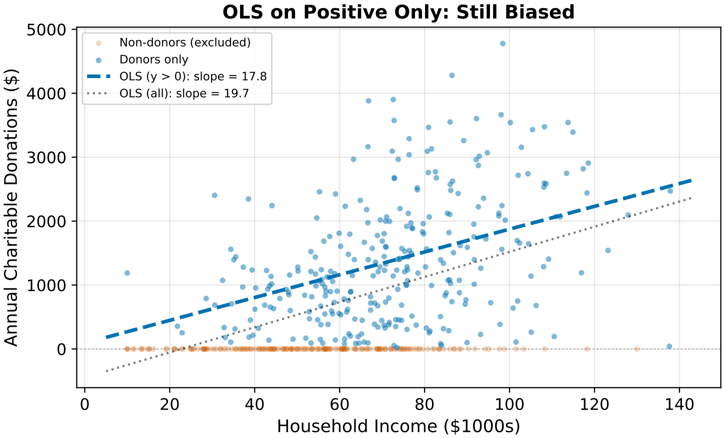

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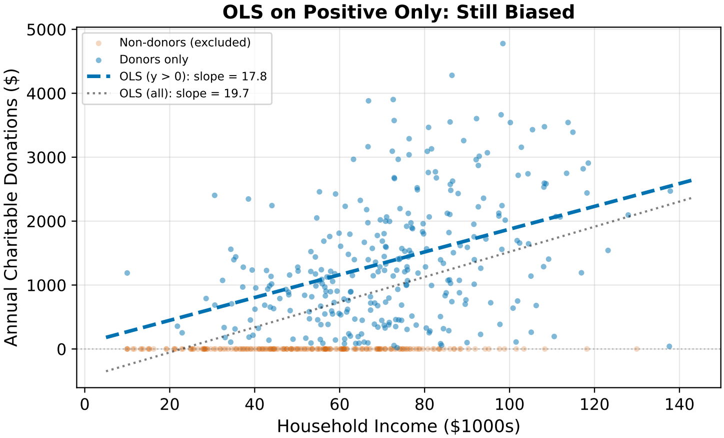


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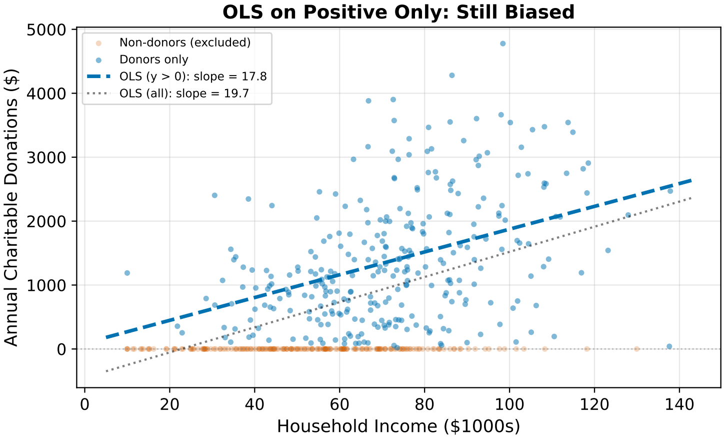


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**The problem:** by conditioning on $y > 0$, we have selected a non-random subsample. Among

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$\Longrightarrow$ Both approaches ignore the **mechanism** that generates the zeros. We need a model that understands *why* some households donate zero.

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$\Longrightarrow$ We need a model that distinguishes between the **desired** outcome and the **observed** outcome. This is the latent variable idea you already know from logit/probit.

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$\Longrightarrow$ The Tobit model reconstructs those unobserved desires. It posits a **latent variable** for what each household *wants* to give, and an **observation rule** that censors negative desires to zero.

# Latent Variable Framework

Define the **latent (unobserved)** variable $y_i^*$ as each household's *desired* donations:

$$y_i^* = \beta_0 + \beta_1 \,\text{Income}_i + \beta_2 \,\text{Educ}_i + \beta_3 \,\text{Children}_i + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma^2)$$

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$\Longrightarrow$ The zeros are not real zeros. They are **censored observations** where the true desired donation is negative.

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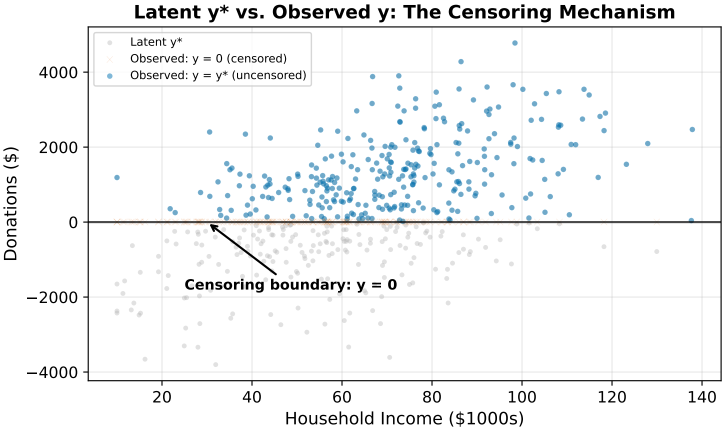

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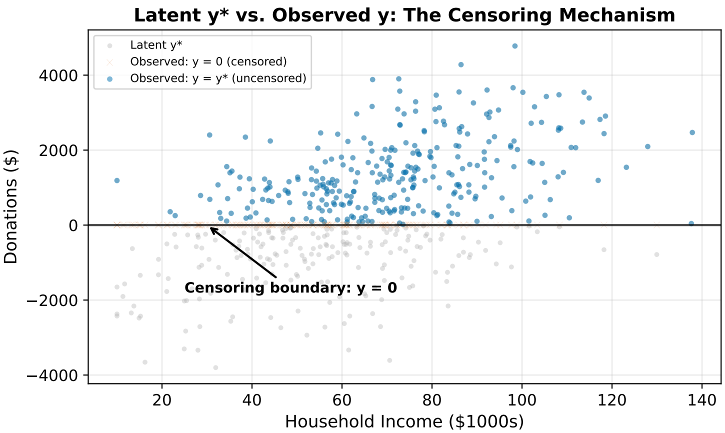


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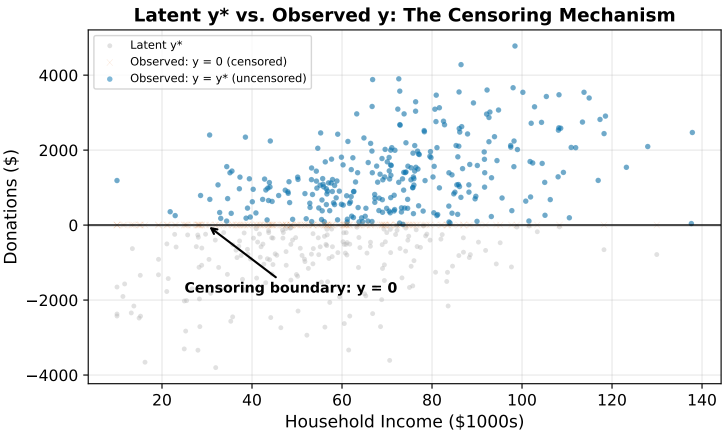


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$$f(y_i) = \frac{1}{\sigma}\,\phi\left(\frac{y_i - \beta_0 - \beta_1 \text{Income}_i - \beta_2 \text{Educ}_i - \beta_3 \text{Children}_i}{\sigma}\right)$$

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Why the negative sign inside $\Phi$? We need $P(\varepsilon_i \leq -\text{XB}_i)$. Flipping the sign converts $y_i^* \leq 0$ into a standard CDF evaluation.

# The Log-Likelihood

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$\Longrightarrow$ The Tobit likelihood combines a regression component and a binary component into a single model.

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| $\hat{\beta}_1$ (Income, per \$1000) | 32.3 | 30 |
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$\Longrightarrow$ By modeling the censoring mechanism explicitly, Tobit recovers the **latent** slope that OLS cannot.

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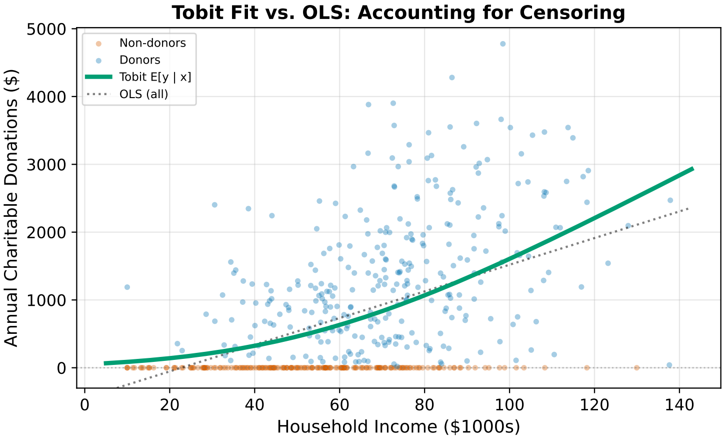

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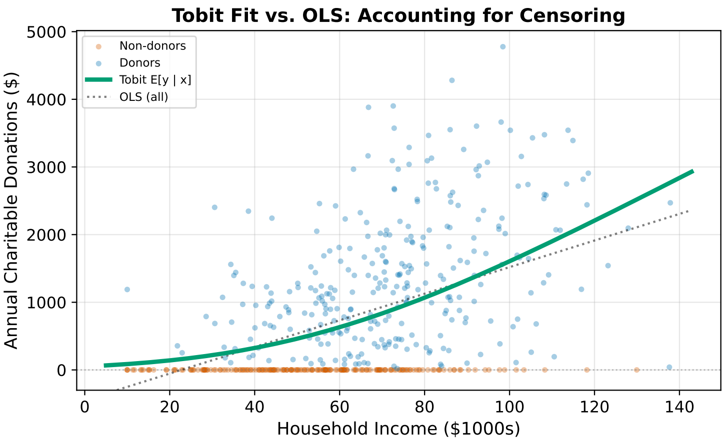


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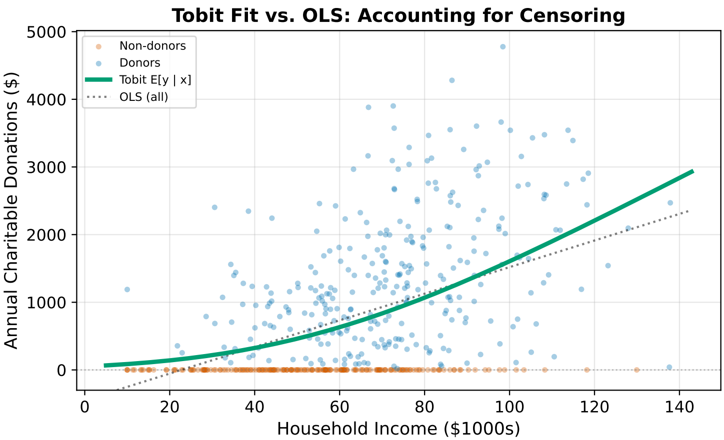


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Notice the curve is flat near zero for low incomes (most households are censored, so additional income

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In OLS, $\hat{\beta}_1$ is the marginal effect, full stop. In Tobit, $\hat{\beta}_1 = 32.3$ is the effect on the **latent** variable $y^*$. But we observe $y = \max(0, y^*)$, so different research questions call for different marginal effects:

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2. **"Does an extra dollar of income bring new households into the donor pool?"** The effect on $P(y > 0)$, the extensive margin. Use this when the policy goal is to increase *participation*

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2. **"Does an extra dollar of income bring new households into the donor pool?"** The effect on $P(y > 0)$, the extensive margin. Use this when the policy goal is to increase *participation*
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$\Longrightarrow$ Each question has its own formula. Let's see them.

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**Use this when** you are interested in the structural relationship between income and the desire to donate, ignoring the censoring constraint. This is the parameter OLS was trying (and failing) to estimate.

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$\Longrightarrow$ Effects 2 and 3 depend on *where* you evaluate them (which household). A household far from the censoring threshold has different marginal effects than one near it.

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$\Longrightarrow$ For this household (only 28% chance of donating), most of the income effect is "absorbed" by the probability margin. The unconditional effect (\$9.1) is far below the latent effect (\$32.3).

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**High-income household** (Income $= 100$, Educ $= 16$, Children $= 1$):

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$\Longrightarrow$ This household is nearly certain to donate (92%), so almost all of the latent effect passes through to the observed outcome. The unconditional ME (\$29.9) is close to the raw coefficient (\$32.3).

# Comparing Marginal Effects Across Households

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$\Longrightarrow$ Where a household sits relative to the censoring threshold determines which margin dominates.

# Boundary Condition: When Tobit Reduces to OLS

Look at the unconditional marginal effect formula again:

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$\Longrightarrow$ If censoring is rare (few zeros), Tobit and OLS give similar answers. The more censoring, the more Tobit differs from OLS.

# The McDonald-Moffitt Decomposition

A policymaker wants to increase total charitable giving. Should they target existing donors to give more (intensive margin), or convert non-donors into donors (extensive margin)? The McDonald-Moffitt decomposition answers exactly this.

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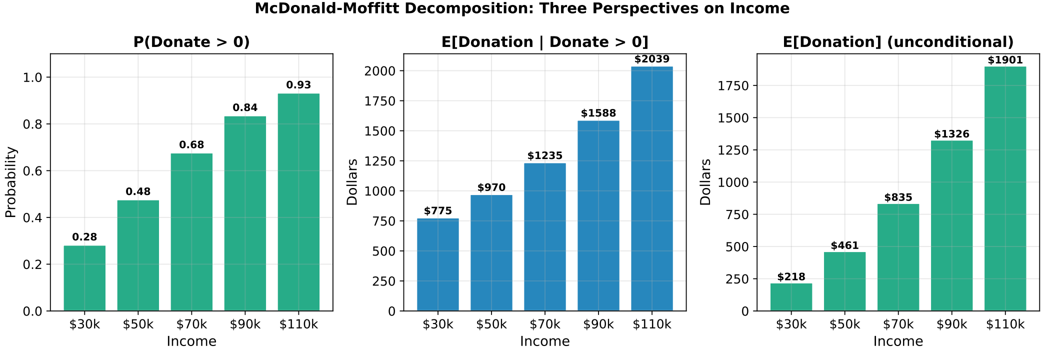
$\Longrightarrow$ A single coefficient $\beta_1$ drives *both* the probability of participation and the level of giving. This is the defining feature (and restriction) of the Tobit model.

# Visualizing the Decomposition



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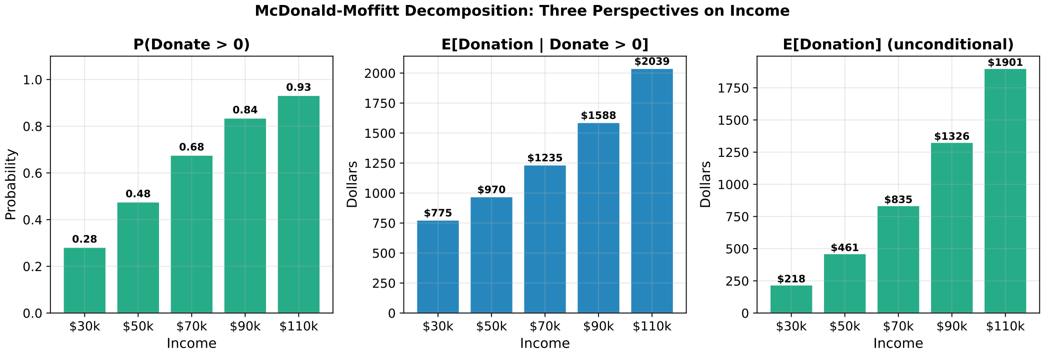


At Income = \$30k, only 28% donate; at \$110k, 93% donate. Among donors, average giving rises from \$775 to \$2,039. Both channels contribute to unconditional E[$y$] rising from \$218 to \$1,901.

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$\Longrightarrow$ With censoring, the zeros are **in the data**. With truncation, the zeros are **missing entirely**.

# When Do You Face Each Situation?

**Censored examples** (use Tobit):

- Charitable donations: non-donors report \$0, all households surveyed
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$\Longrightarrow$ When Assumption 2 fails, the Tobit model forces a single coefficient to represent two distinct processes. What alternatives exist?

# When Assumption 2 Fails: Alternatives

**Two-Part Model** (also called the “hurdle” model):

- Part 1: Probit or logit for $P(y > 0)$ with coefficients $\gamma$
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- Adds a selection correction (inverse Mills ratio) to the outcome equation

$\Longrightarrow$ Tobit assumes one mechanism. The two-part model relaxes that. The Heckman model is for a fundamentally different problem: selection, not censoring.

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4. **Is your outcome a count?** (Non-negative integers: 0, 1, 2, . . . )
   - $\Longrightarrow$ **Poisson / Negative Binomial**, not Tobit

# Outline

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5. **The McDonald-Moffitt decomposition** splits the total effect into a participation channel (new donors) and an amount channel (existing donors give more)
6. **Tobit assumes one mechanism** for participation and amount. When that fails, use a two-part model. When zeros come from selection rather than censoring, use Heckman

# Comparison Table: Tobit vs. Alternatives

| | **Tobit** | **Two-Part** | **Heckman** |
|---|---|---|---|
| Zero mechanism | corner solution | corner solution | selection |
| Same $\beta$ for participation & amount? | yes | no (separate $\gamma$, $\delta$) | no (separate equations) |
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$\Longrightarrow$ The correct choice depends on the economic mechanism generating the zeros: censoring from a corner solution, or selection from a missing-data process.

Thank you!

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