Ridge, Lasso, and Elastic Net

More Is Not Always Merrier

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Outline

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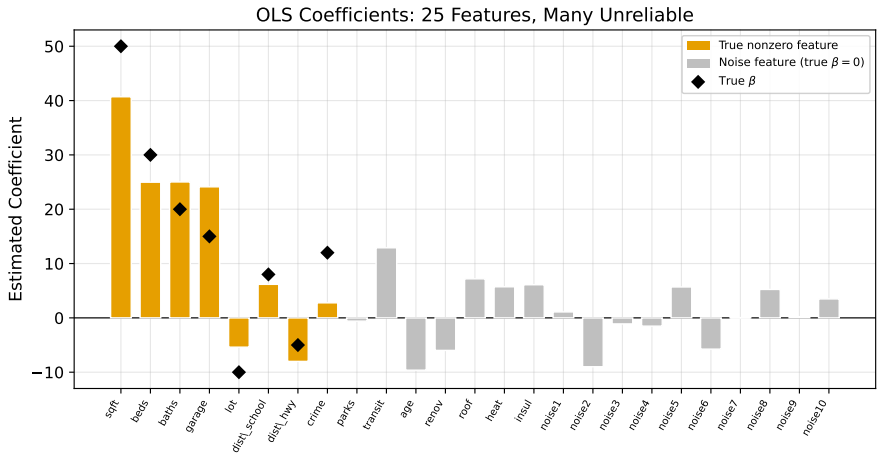
What went wrong?

OLS with 25 Features: Overfitting



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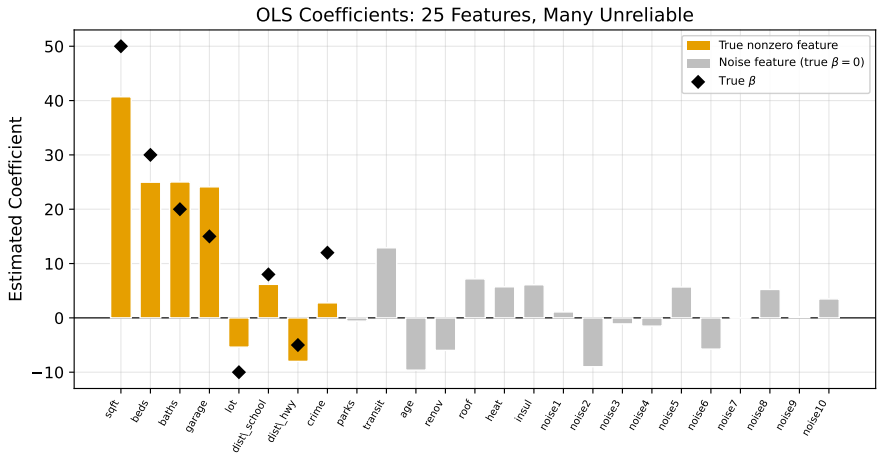

OLS assigns large coefficients to many features. Some of these estimated effects are real. Others are noise that OLS is fitting by mistake. But from the OLS output alone, you cannot tell which is which.

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$\Longrightarrow$ We need a method that deliberately sacrifices some in-sample fit to gain better out-of-sample prediction.

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Size (5 features)	sqft, beds, baths, garage, lot	$\rho = 0.7$	5 of 5
Location (5 features)	dist to school, hwy, crime, parks, transit	$\rho = 0.6$	3 of 5
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$\Longrightarrow$ OLS failed because it treated noise features as if they were real, and multicollinearity made even the real estimates unreliable.

A Different Goal: Prediction, Not Causation

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$\Longrightarrow$ Regularization is a prediction tool. It deliberately introduces bias to reduce variance.

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What kind of penalty should we use? The three main choices: Ridge (ℓ_2), Lasso (ℓ_1), and Elastic Net (both). The subscript tells you the exponent: ℓ_2 squares each coefficient, ℓ_1 takes absolute values.

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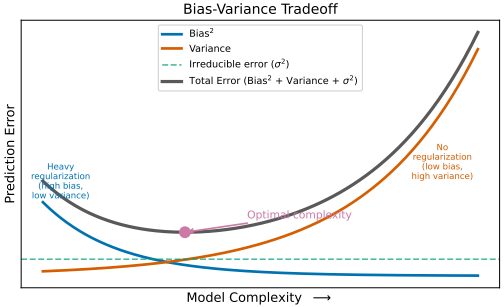

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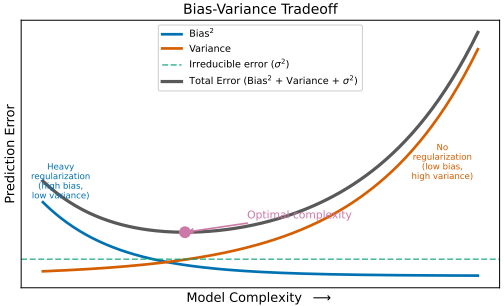

The dashed line is the irreducible error (σ^2); the goal is the complexity level that minimizes total error.

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Note: we standardize all features before fitting (subtract mean, divide by SD). This ensures the penalty treats all features equally, regardless of their original scale. The intercept is not penalized.

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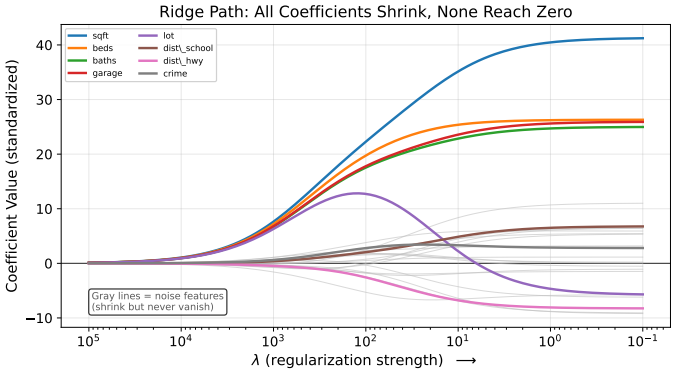
$\implies$ Ridge always keeps all features in the model. It reduces their influence but does not remove them.

Ridge Coefficient Path



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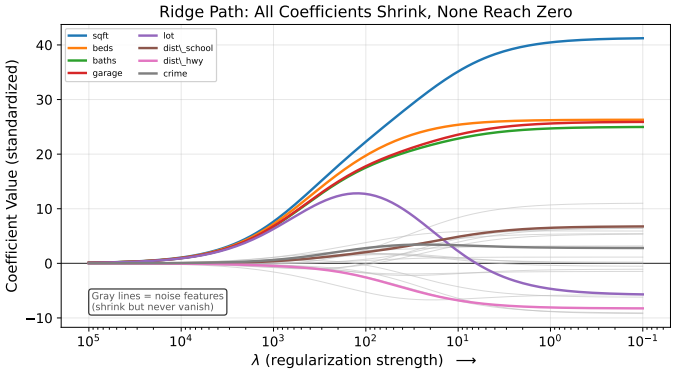

As λ increases (left to right), every coefficient shrinks toward zero. The noise features (gray) become negligible, but no coefficient ever reaches exactly zero.

Ridge Properties

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$\implies$ Ridge is a workhorse for prediction with correlated features, but it does not tell you which features to drop.

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All we need to change is the shape of the penalty.

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- The ℓ_2 penalty (Ridge) costs more as $|\beta_j|$ grows, so it pushes large coefficients hard but barely touches small ones.
- The ℓ_1 penalty (Lasso) charges a **constant rate** per unit of $|\beta_j|$, so it pushes small coefficients all the way to zero. If $\beta = 0.1$, the ℓ_2 cost is 0.01 (negligible) but the ℓ_1 cost is 0.1 (same rate as for $\beta = 10$).

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$\implies$ Lasso performs **feature selection**: it sets some $\hat{\beta}_j$ to exactly zero.

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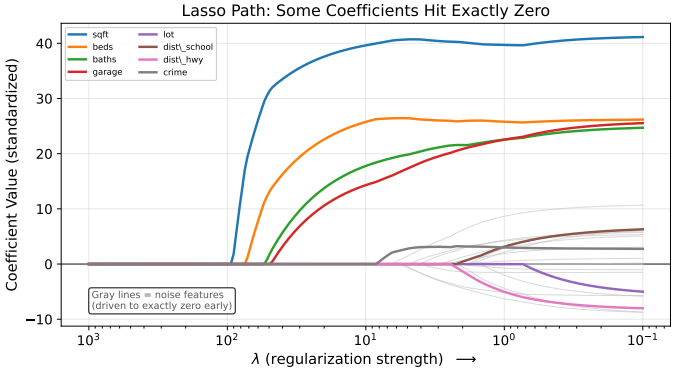

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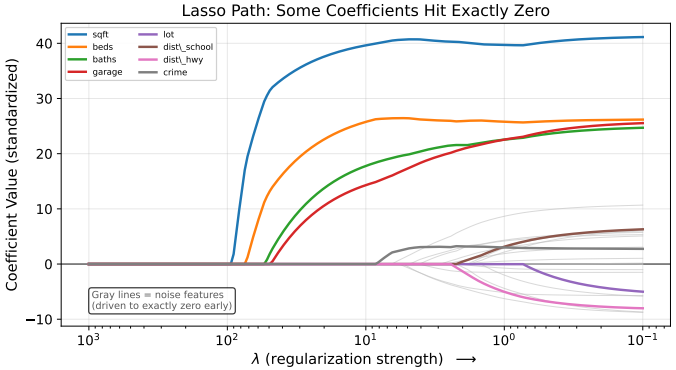
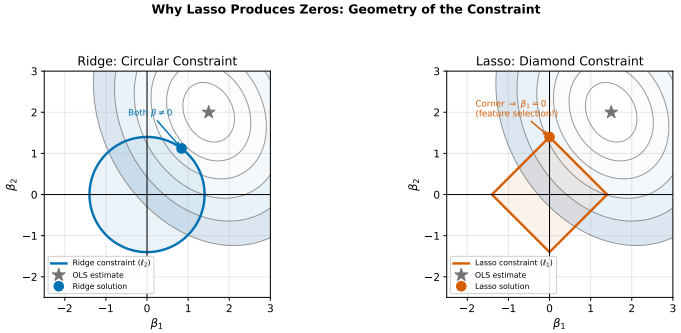


As λ increases, noise features (gray) are driven to zero **first**. The true signal features persist longer. At the right level of λ, only the genuinely predictive features remain.

Why Does Lasso Produce Zeros? The Geometry



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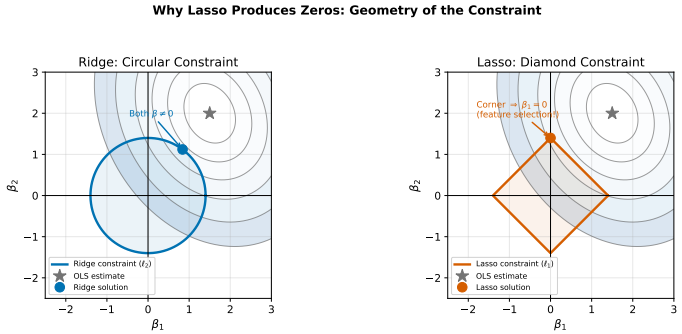

Each ellipse is a set of (β_1, β_2) pairs with the same RSS. As we shrink the constraint region, the first contact point is the solution. Ridge uses a **circle** ($\beta_1^2 + \beta_2^2 \leq t$); Lasso uses a **diamond** ($|\beta_1| + |\beta_2| \leq t$).

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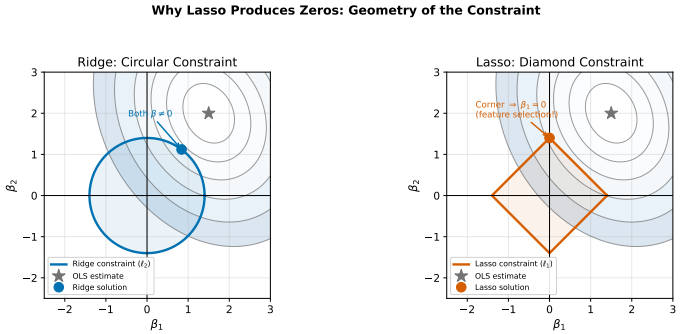

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$\implies$ Diamonds have corners that jut out along the axes, so contact tends to happen at a corner, setting one or more $\beta_j = 0$.

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Compare the approaches:

Method	**Features Retained**	**Zeros?**
OLS	All 25	No
Ridge	All 25 (small)	No
Lasso	$\sim$10–12	Yes

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In our house price example, the Lasso with a well-chosen λ retains about a dozen features and sets the rest to zero.

Compare the approaches:

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$\Longrightarrow$ Lasso serves double duty: it improves prediction *and* identifies which variables to keep.

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$\Longrightarrow$ When features come in correlated groups (as in our house data), Lasso's selection can be unstable. This motivates combining the two penalties.

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$\Longrightarrow$ Combine the ℓ_1 and ℓ_2 penalties into a single objective.

Outline

Elastic Net: Combining ℓ_1 and ℓ_2

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Two tuning parameters:

- $\lambda \geq 0$: overall regularization strength (same role as before)
- $\alpha \in [0, 1]$: the **mixing parameter** between Lasso ($\alpha = 1$) and Ridge ($\alpha = 0$)

Note: this α is unrelated to the significance level from hypothesis testing. The $\frac{1}{2}$ in the ℓ_2 component is a standard convention; since λ is chosen by CV, this scaling is absorbed into tuning.

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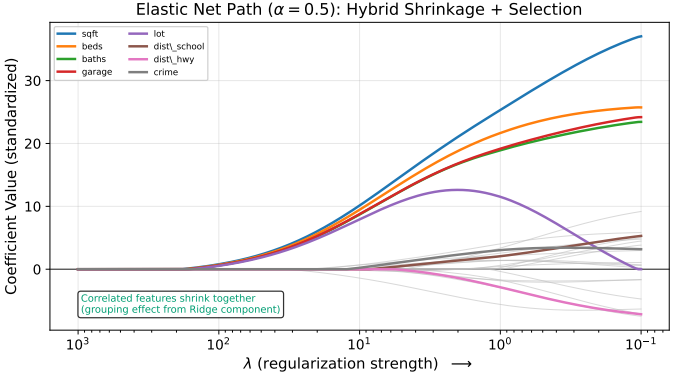
α	**Method**	**Behavior**
$\alpha = 1$	Lasso	Feature selection, no grouping
$\alpha = 0$	Ridge	Shrinkage only, no selection
$0 < \alpha < 1$	Elastic Net	Selection + grouping

Elastic Net Coefficient Path



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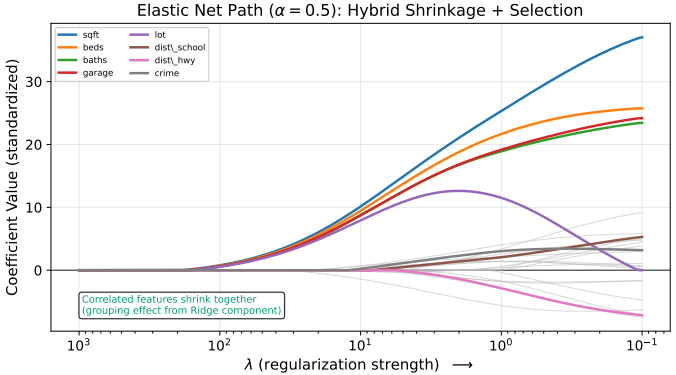

Like Lasso, noise features are driven to zero. Like Ridge, correlated features within the same block shrink **together** rather than one being arbitrarily dropped. This is the **grouping effect**.

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$\Longrightarrow$ Elastic Net inherits the best of both worlds: sparsity from Lasso and stability from Ridge.

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How Do We Choose λ?

Now that we have three methods (Ridge, Lasso, Elastic Net), each requires choosing λ. Too small $\implies$ overfitting. Too large $\implies$ underfitting.

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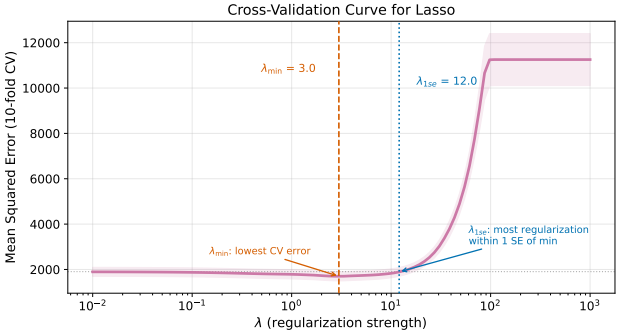
$\implies$ Cross-validation simulates out-of-sample prediction using only the training data. It selects λ without touching the test set.

The CV Curve: λ_{min} and λ_{1se}



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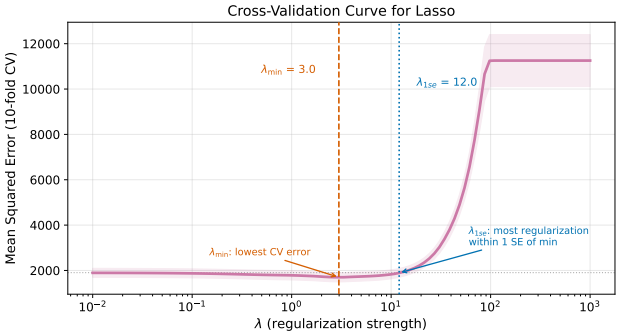

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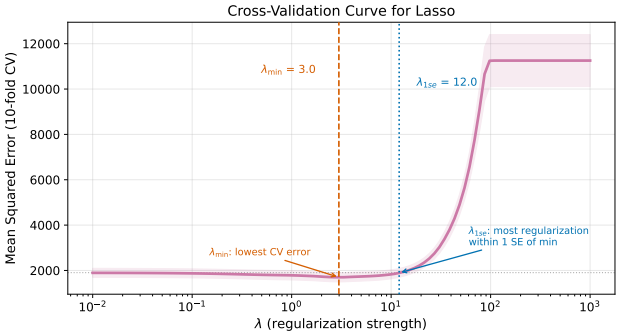

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$\Longrightarrow$ λ_{1se} is the "one-standard-error rule": when two models predict nearly equally well, prefer the simpler one.

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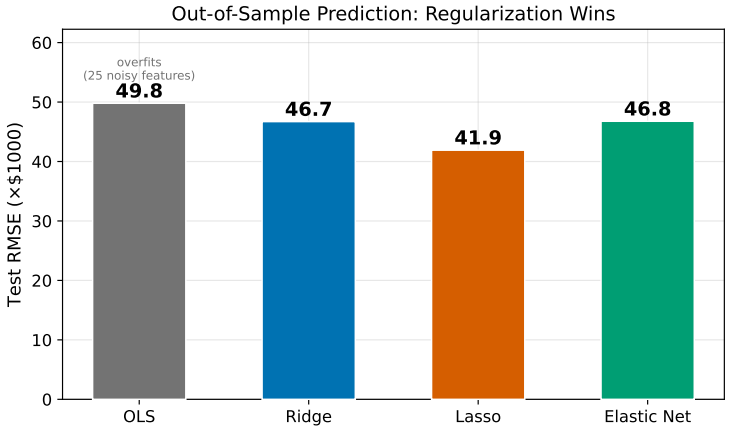
Prediction Comparison: Test RMSE



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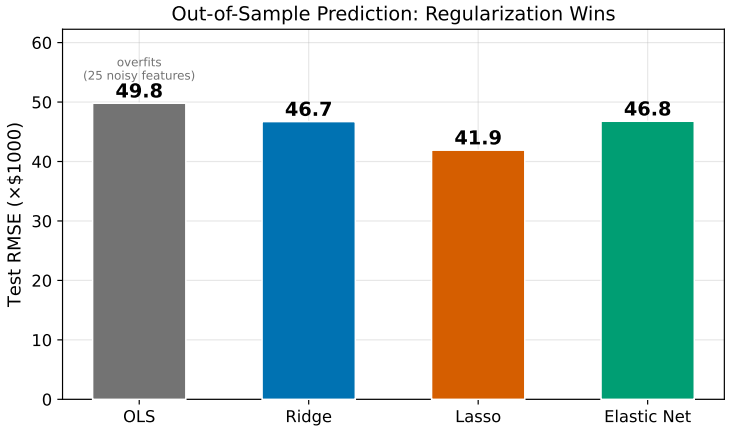

All three regularized methods beat OLS on out-of-sample prediction. Ridge, Lasso, and Elastic Net perform similarly here; the improvement comes from penalizing noise features.

Ridge vs. Lasso vs. Elastic Net: When to Use Which

Scenario	**Ridge**	**Lasso**	**EN**
Many correlated features	✓		✓
Want feature selection		✓	✓
Few true signals, many noise		✓	✓
$p > n$	✓	(limited)	✓
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$\implies$ In all cases, choose λ by cross-validation. Never choose λ by hand or by in-sample fit.

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$\Longrightarrow$ Use regularization for prediction. Use IV, FE, or other causal methods for inference. Do not mix them up.

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$\Longrightarrow$ Regularization is a prediction tool: it trades bias for lower variance. For causal inference, use IV, FE, or RE instead.

Thank you!

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