Time Series: Dynamic Models and Autocorrelation

Modeling Temporal Dependence in Economic Data

Jake Anderson

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Outline

Cross-Section vs Time Series

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$\Longrightarrow$ The i.i.d. assumption fails. We need new tools that account for **temporal dependence**.

Consequences of Temporal Dependence

When observations are correlated over time:

1. Standard OLS **standard errors are wrong** (usually too small)
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This lecture covers:

- How to characterize temporal dependence (stationarity, ACF)
- Models that exploit it (AR, ARDL)
- How to detect it in regression residuals (Breusch-Godfrey)
- How to forecast with it

Covariance Stationarity: Definition

A time series $\{Y_t\}$ is **covariance stationary** if:

1. **Constant mean:** $E(Y_t) = \mu$ for all t
2. **Constant variance:** $\text{Var}(Y_t) = \sigma^2$ for all t
3. **Covariance depends only on distance:** $\text{Cov}(Y_t, Y_{t-k})$ depends only on k, not on t

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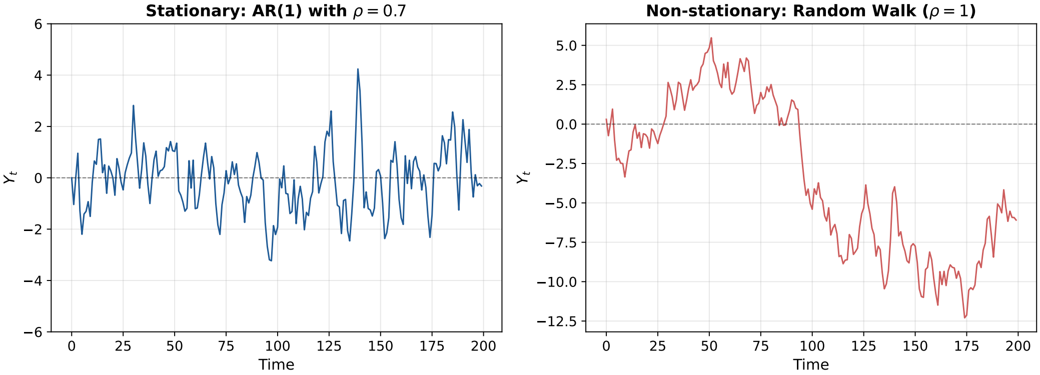
$\Longrightarrow$ Stationarity ensures that estimated statistical properties are meaningful and stable.

Stationary vs Non-stationary: Visual Comparison



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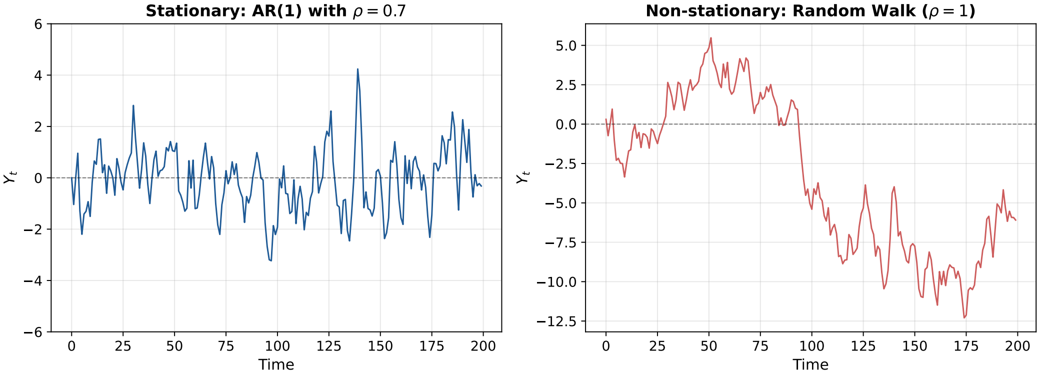

Left: mean-reverting; always pulled back toward zero. **Right:** wanders without bound; no tendency to return.

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Example: Stock prices are approximately random walks. That is why financial economists work with *returns* (which are approximately stationary) rather than price levels.

Measuring Temporal Dependence: The ACF

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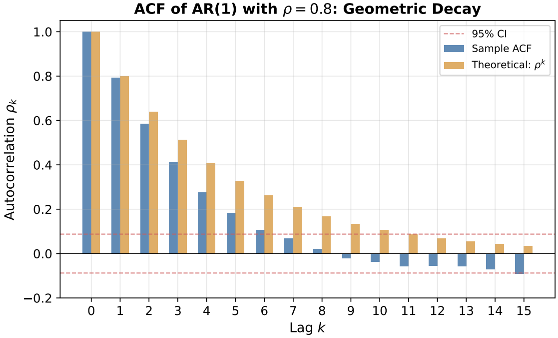
The **sample autocorrelation**:

$$r_k = \frac{\sum_{t=k+1}^{T}(Y_t - \bar{Y})(Y_{t-k} - \bar{Y})}{\sum_{t=1}^{T}(Y_t - \bar{Y})^2}$$

Reading an ACF Plot



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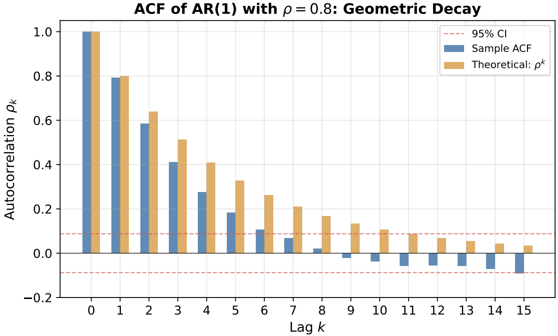

How to read it:

- Bars beyond the dashed lines = statistically significant autocorrelation
- For an AR(1), the ACF decays **geometrically**: $\rho_k = \rho^k$
- If residuals from a model show significant ACF spikes $\implies$ the model is missing dynamics

Autoregressive Models: AR(p)

An **AR(p) model** says today's value depends on its own past p values:

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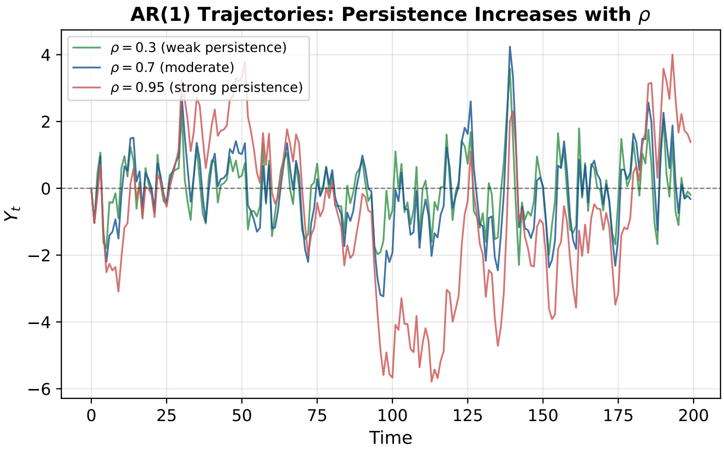
AR(2): $Y_t = \delta + \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + v_t$

- Allows richer dynamics: oscillations, humps

How Persistence Changes with ρ



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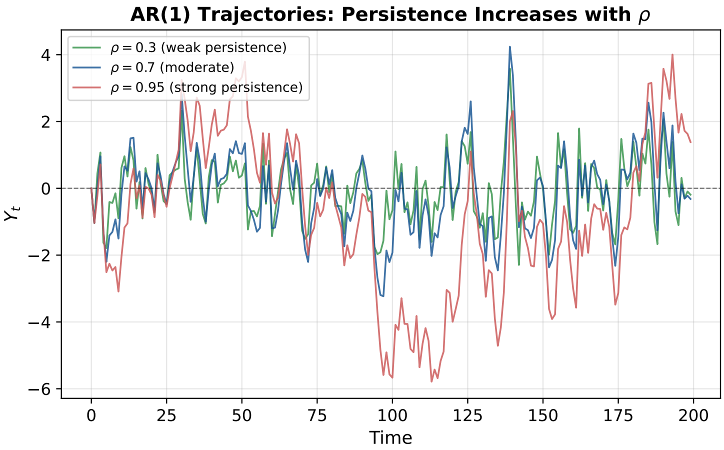

Same shocks, different ρ. Higher $\rho \implies$ slower mean reversion, longer "memory."

Model Selection: AIC and BIC

How many lags? Use **information criteria**:

$$\text{AIC} = \ln(\hat{\sigma}^2) + \frac{2K}{T}, \qquad \text{BIC} = \ln(\hat{\sigma}^2) + \frac{K\ln(T)}{T}$$

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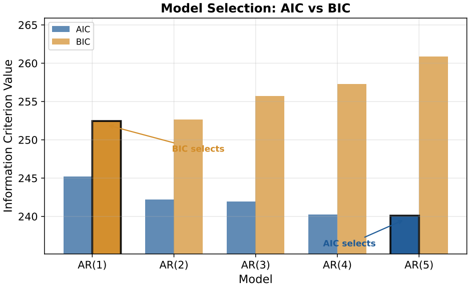

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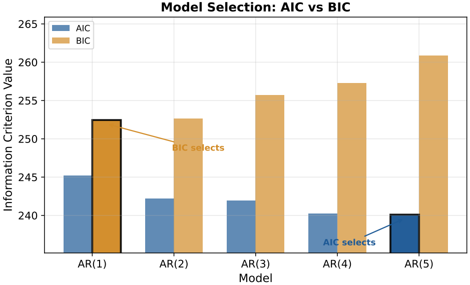

When AIC and BIC disagree, BIC is typically preferred for consistent selection.

Autoregressive Distributed Lag: ARDL(p, q)

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Example: ARDL(1,1) Phillips Curve

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$\implies$ Current inflation depends on: (1) its own recent history, and (2) current *and* past unemployment changes.

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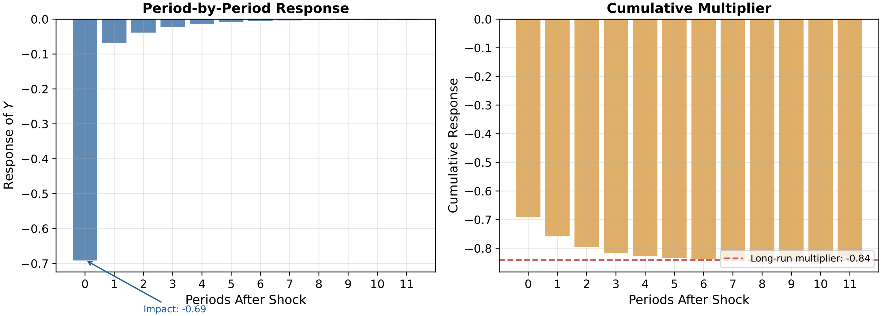
$\Longrightarrow$ The long-run effect is larger (in absolute value) than the impact effect whenever $|\theta_1| > 0$.

Impulse Response: Visualizing the Multipliers

ARDL(1,1) with $\hat{\theta}_1 = 0.56$, $\hat{\delta}_0 = -0.69$, $\hat{\delta}_1 = 0.32$:



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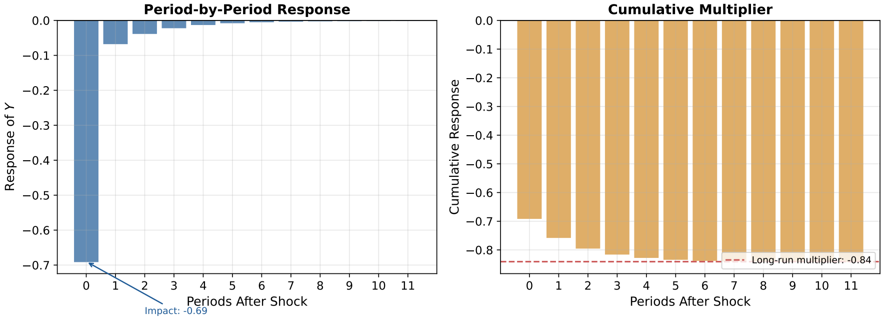

- Impact multiplier: -0.69
- Interim: $-0.69 + 0.32 = -0.37$
- Long-run: $\frac{-0.69+0.32}{1-0.56} = -0.84$

Multiplier Practice Problem

An ARDL(1,1) model for quarterly inflation gives:

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$\implies$ The long-run effect (-0.84) exceeds the impact (-0.69) because the AR term propagates the shock.

Serial Correlation in Regression Residuals

Suppose we estimate:

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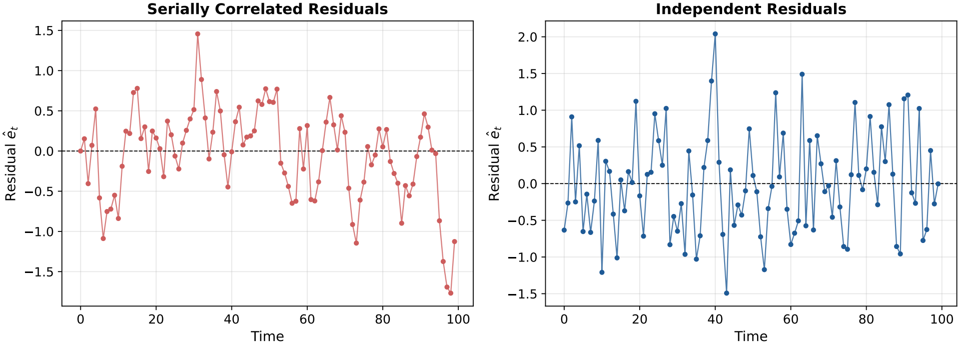
$\implies$ The coefficients are fine, but our **inference** (tests, CIs) is unreliable.

What Serially Correlated Residuals Look Like



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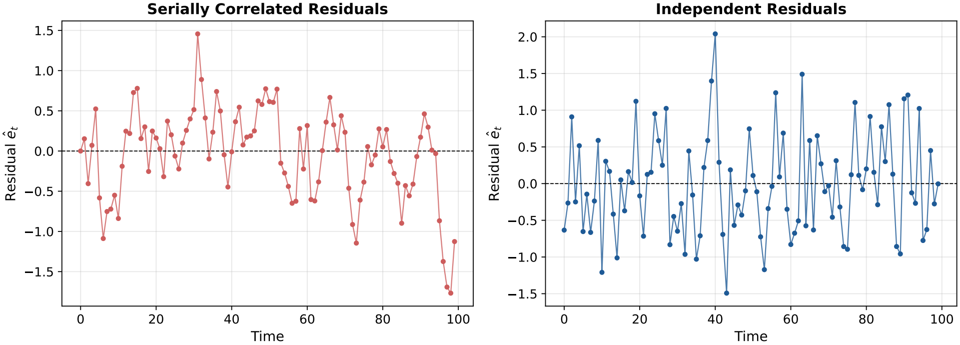

Left: runs of positive then negative residuals $\Longrightarrow$ positive autocorrelation. **Right:** residuals bounce randomly around zero $\Longrightarrow$ no serial correlation.

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Important: the auxiliary regression *includes all original regressors*. This is what makes BG valid even with lagged dependent variables (unlike Durbin-Watson).

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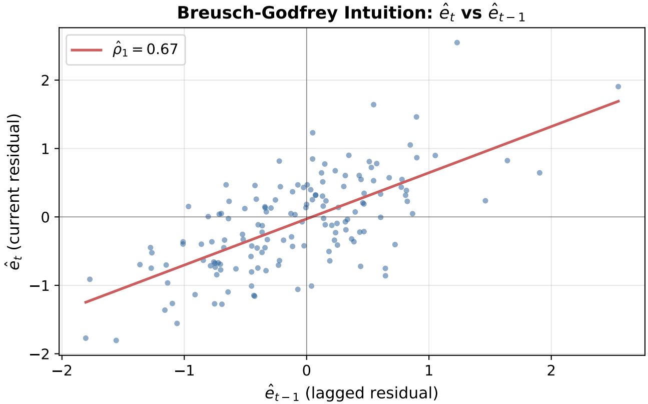

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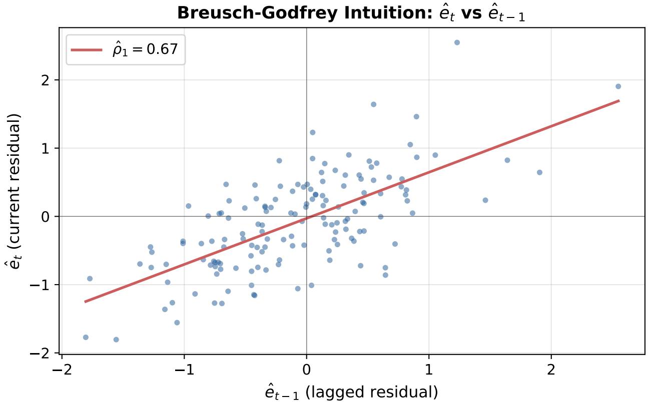

The BG auxiliary regression checks whether $\hat{\rho}_1$ (the slope in this scatter) is significantly different from zero, *after controlling for the original regressors*.

Forecasting with AR Models

One-step-ahead forecast from an AR(2):

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$\Longrightarrow$ Each forecast builds on previous forecasts, compounding uncertainty.

Forecast Practice Problem

An AR(2) model for quarterly inflation gives:

$$\hat{\delta} = 0.4523, \quad \hat{\theta}_1 = 0.6234, \quad \hat{\theta}_2 = 0.2145$$

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Solution:

$$\begin{aligned}\widehat{\text{INF}}_{T+1} &= 0.4523 + 0.6234(2.5) + 0.2145(3.0) \\ &= 0.4523 + 1.5585 + 0.6435 \\ &= 2.65\end{aligned}$$

Forecast Intervals

As we forecast further ahead, uncertainty grows:

$$\hat{Y}_{T+h} \pm t_c \cdot \hat{\sigma}_h$$

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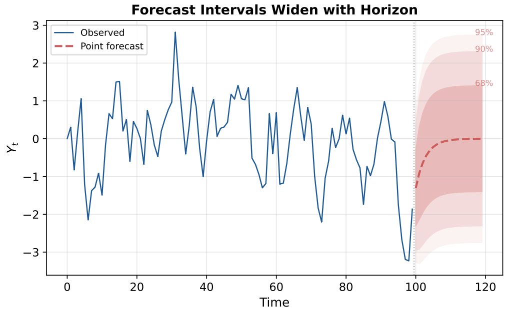

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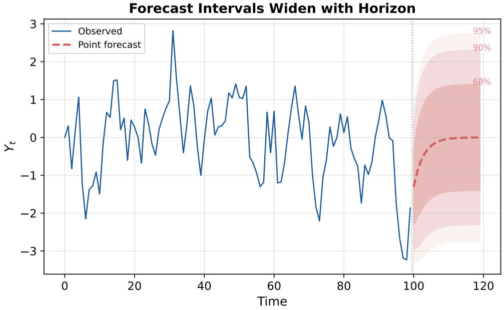

$\Longrightarrow$ Short-horizon forecasts are relatively precise. Long-horizon forecasts revert toward the unconditional mean with wide bands.

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In-sample: fit the model on all available data, check residuals.

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- **RMSE** $= \sqrt{\frac{1}{P}\sum_{h=1}^{P}(\hat{Y}_{T+h} - Y_{T+h})^2}$
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$\Longrightarrow$ Always evaluate forecasts out-of-sample when the goal is prediction.

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7. **Forecasting:** iterate AR/ARDL forward; confidence bands widen with horizon. Evaluate out-of-sample.

Thank you!

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