

The Hausman-Taylor Estimator

Estimating Time-Invariant Effects with Endogeneity

Jake Anderson

May 16, 2026

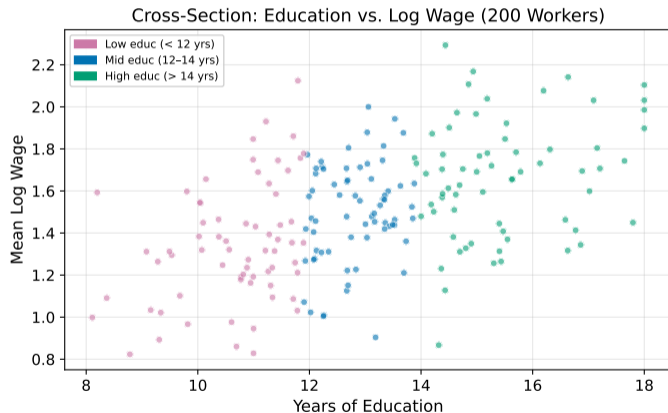
- 1 The Problem: Returns to Education
- 2 The Hausman-Taylor Estimator
- 3 When HT Works (and When It Doesn't)
- 4 Decision Framework
- 5 Summary

The Data

A policymaker wants to know the return to one more year of schooling. We have panel data on **200 workers** over **5 years**. How hard can it be?

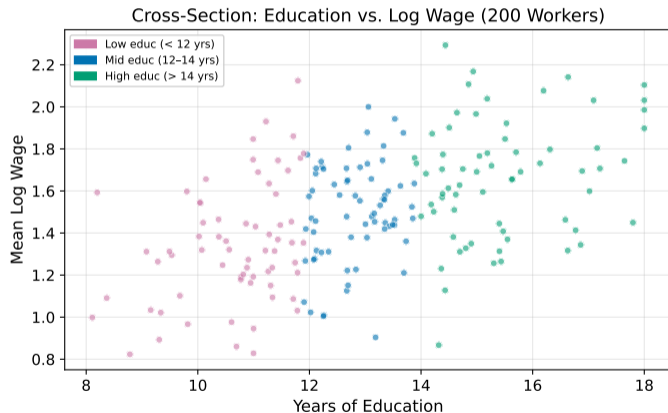
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Higher education is associated with higher wages. But is the relationship **causal**?

Pooled OLS: A First Pass

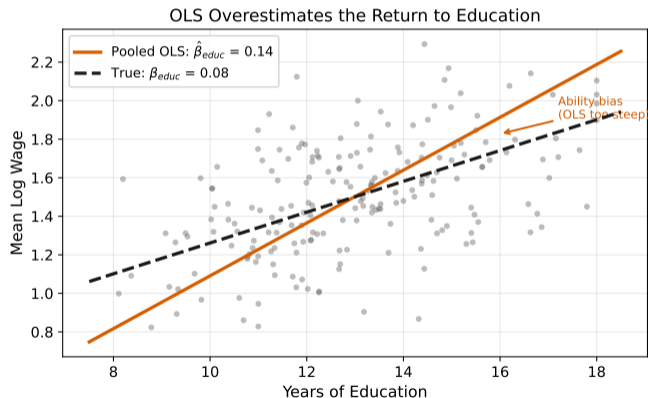
Run pooled OLS of log wages on education, experience, union, and race:

$$\log(\text{wage}_{it}) = \beta_0 + \beta_1 \text{educ}_i + \beta_2 \text{exper}_{it} + \beta_3 \text{union}_{it} + \beta_4 \text{black}_i + \varepsilon_{it}$$

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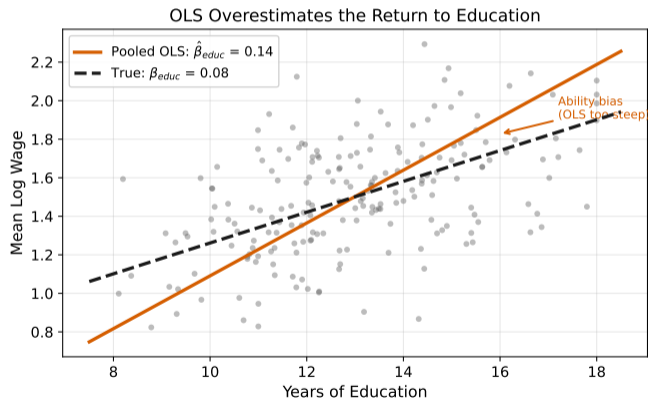
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OLS estimates $\hat{\beta}_{educ} \approx 0.14$, but the true return is 0.08. Why the overestimate?

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OVB formula (single-regressor version):

$$\hat{\beta}_{\text{educ}}^{\text{OLS}} = \underbrace{\beta_{\text{educ}}}_{\text{true}} + \underbrace{\beta_{\alpha}}_{\text{effect of ability on wages}} \times \underbrace{\delta_1}_{\text{relationship of ability to educ}}$$

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Both $\beta_{\alpha} > 0$ and $\delta_1 > 0 \implies$ OLS is **biased upward**.

Fixed Effects to the Rescue?

We know FE eliminates time-invariant unobservables. Write the model as:

$$\log(\text{wage}_{it}) = \alpha_i + \beta_1 \text{educ}_i + \beta_2 \text{exper}_{it} + \beta_3 \text{union}_{it} + \beta_4 \text{black}_i + \varepsilon_{it}$$

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$$\log(\text{wage}_{it}) - \overline{\log(\text{wage})}_i = \beta_2(\text{exper}_{it} - \overline{\text{exper}}_i) + \beta_3(\text{union}_{it} - \overline{\text{union}}_i) + (\varepsilon_{it} - \overline{\varepsilon}_i)$$

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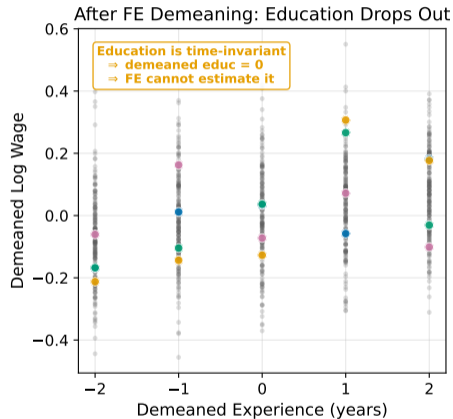
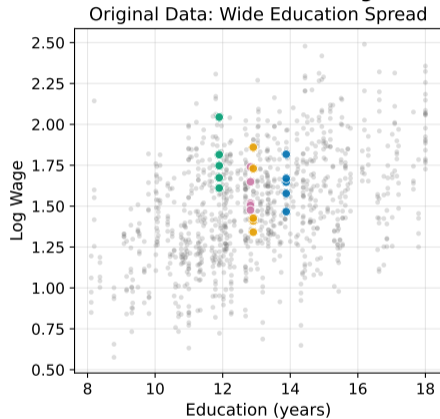
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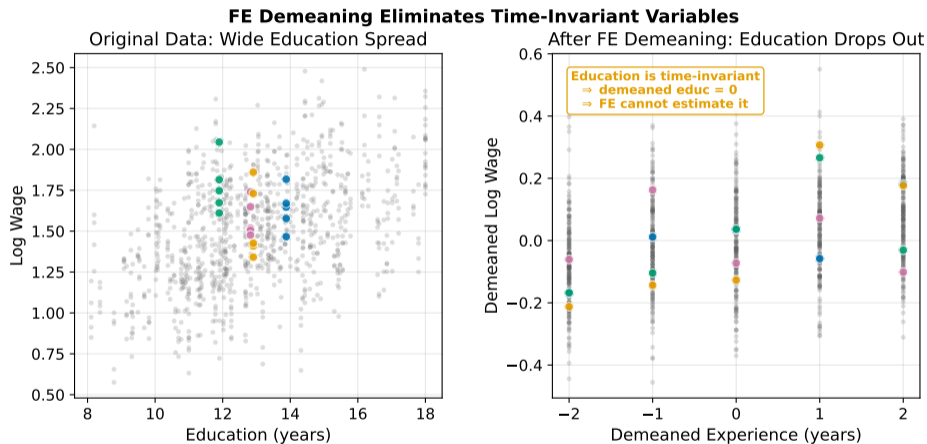
⇒ **Education is time-invariant.** Demeaning removes it entirely.

FE Demeaning: Education Disappears

FE Demeaning Eliminates Time-Invariant Variables



FE Demeaning: Education Disappears



Each worker has **one education level** across all 5 years. After demeaning, the within-worker variation in education is exactly zero. FE cannot estimate β_{educ} .

Random Effects: Can It Estimate Education?

RE treats α_i as a random draw, uncorrelated with the regressors:

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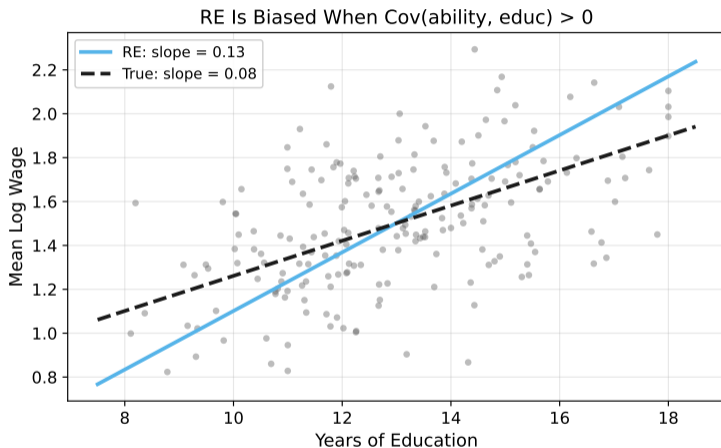
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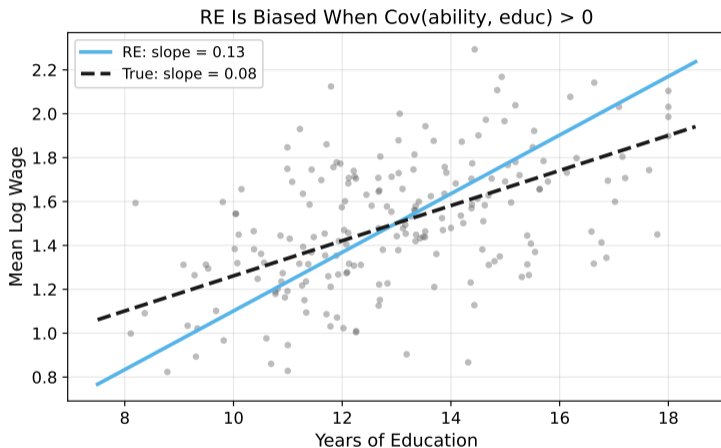
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We just argued this is **violated** (ability correlates with education). So the RE estimate is **biased**.





RE estimates $\hat{\beta}_{\text{educ}} \approx 0.13$ (true = 0.08). The ability bias persists because RE does not fully eliminate α_i from the estimation.

The Dilemma

Abbreviations: TV = time-varying; TI = time-invariant.

Method	Estimate $\hat{\beta}_{\text{educ}}$	Consistent?	Problem
OLS	≈ 0.14	No	Omits ability
FE	N/A		Cannot estimate TI vars
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⇒ We need an estimator that handles **time-invariant endogenous** variables.

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\implies We already have instruments sitting inside the panel. We just need a procedure that uses them.

Outline

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The insight:

- Some time-varying variables (like experience) are **exogenous**: $\text{Cov}(\alpha_i, \text{exper}_{it}) = 0$
- Their **within-group means** ($\text{ex}\bar{\text{per}}_i$) vary across individuals
- These means are correlated with education (workers who studied longer entered the workforce later \implies less experience)
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The Hausman-Taylor Idea

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\implies The within-group means of time-varying exogenous variables are **valid instruments** for education.

Variable Classification: Four Categories

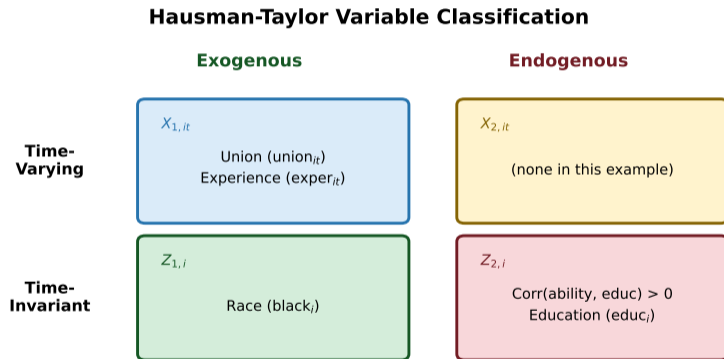
HT requires classifying every variable into one of four groups:

Hausman-Taylor Variable Classification

	Exogenous	Endogenous
Time-Varying	$X_{1,it}$ Union ($union_{it}$) Experience ($exper_{it}$)	$X_{2,it}$ (none in this example)
Time-Invariant	$Z_{1,i}$ Race ($black_i$)	$Z_{2,i}$ Corr(ability, educ) > 0 Education ($educ_i$)

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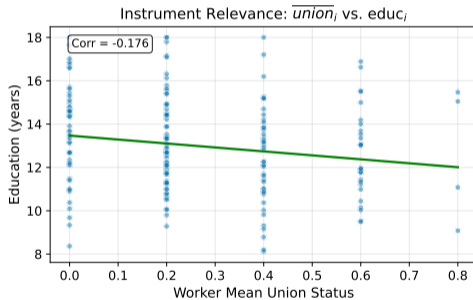
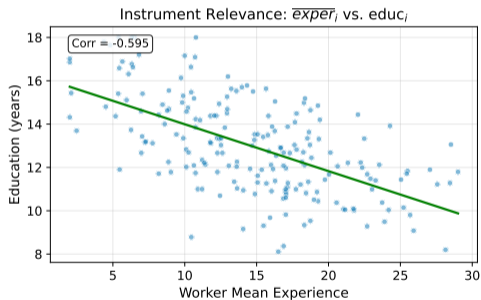


The classification determines which variables need instruments and which variables *provide* instruments.

The Instruments: Within-Group Means

Why do \overline{exper}_i and \overline{union}_i work as instruments for $educ_i$?

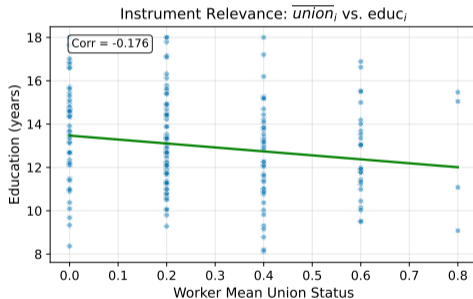
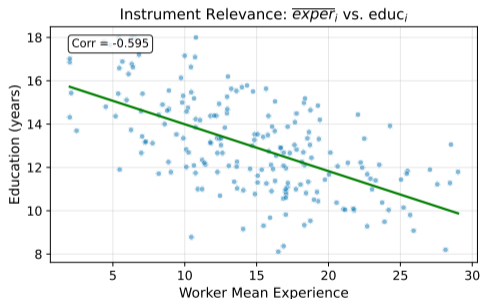
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Within-Group Means of TV Exogenous Variables as Instruments



Relevance: Workers with more education have less experience (delayed entry) and lower unionization.

Validity: These means are uncorrelated with unobserved ability (exogenous by classification).

Instrument Requirements

Recall: X_1 = TV exogenous (exper, union); Z_2 = TI endogenous (educ).

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2. Validity (exclusion restriction):

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This is an **assumption**, not something we can verify directly. It follows from classifying $X_{1,it}$ as exogenous: if $\text{Cov}(\alpha_i, \text{exper}_{it}) = 0$ for all t , then $\text{Cov}(\alpha_i, \text{exper}_i) = 0$.

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When might this fail? If high-ability workers accumulate more experience through promotions or job mobility, then $\text{Cov}(\alpha_i, \text{exper}_{it}) \neq 0$ and the instruments are invalid. The maintained assumption in this example rules that out: experience here reflects labor market entry timing, not ability-driven career advancement.

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\implies Same logic as any IV estimation, but the instruments come from **within the panel**.

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Run FE on the full model. FE wipes out both α_i (good) and the time-invariant regressors (the cost):

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\implies We now have consistent $\hat{\beta}_{\text{exper}}^{FE}$ and $\hat{\beta}_{\text{union}}^{FE}$. We still need $\hat{\beta}_{\text{educ}}$ and $\hat{\beta}_{\text{black}}$.

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⇒ OLS on this equation would still be biased. We need instruments.

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Why does this work?

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HT Procedure: Step 2b (IV Estimation)

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⇒ 2SLS isolates the variation in education that is driven by experience and union patterns, not by ability.

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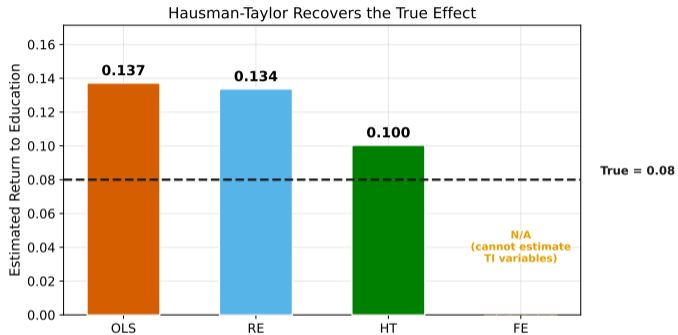
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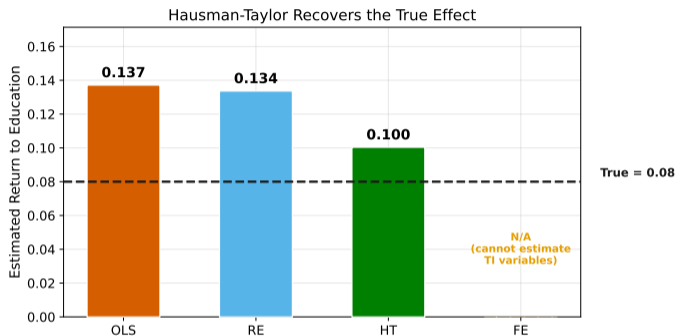
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⇒ You specify which variables are time-invariant and which are endogenous. The estimator does the rest.

The Result: HT Recovers the True Effect

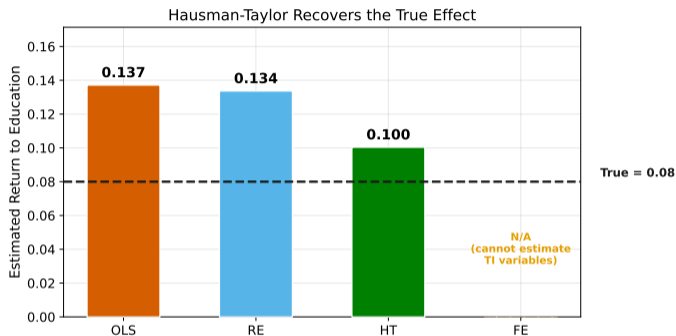


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\implies By instrumenting education with within-group means of experience and union status, HT removes most of the ability bias.

Note: the HT estimate shown is from the IV-between step (Step 2b). The full FGLS estimator (Step 3) produces a slightly different number; Stata's `xthtaylor` implements the complete procedure.

How Each HT Component Solves a Problem

Problem	HT Component	Result
TV coefficients biased	FE (within estimation)	Consistent $\hat{\beta}_{\text{exper}}, \hat{\beta}_{\text{union}}$
TI endogenous variable	IV using $\bar{X}_{1,i}$ as instruments	Consistent $\hat{\beta}_{\text{educ}}$
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⇒ HT fills the gap between FE (drops TI variables) and RE (requires full exogeneity).

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In our example: 2 TV exogenous (exper, union) instrumenting 1 TI endogenous (educ) \implies overidentified (good: we can test validity).

Formal Assumptions

Partition the variables:

- $X_{1,it}$: time-varying, exogenous (exper, union)
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The idiosyncratic error is pure noise.

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\implies Assumption 2 is what makes $\bar{X}_{1,i}$ a valid instrument: it inherits exogeneity from $X_{1,it}$.

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⇒ Rejection suggests that at least one of experience or union is not truly exogenous to ability. This would invalidate the HT approach.

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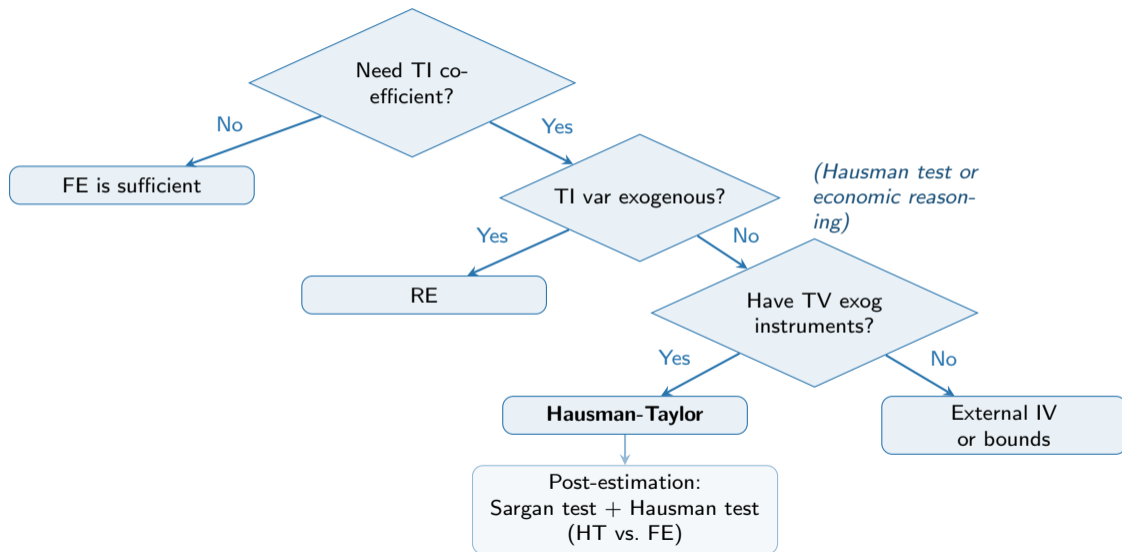
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⇒ This tests whether the TV variables you classified as exogenous truly are. It does *not* test the validity of the TI classification.

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Decision Flowchart



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⇒ When you need a time-invariant coefficient but suspect endogeneity, check whether your TV exogenous variables can serve as internal instruments.

Thank you!

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