Introduction to Random Effects

From Fixed Parameters to Random Draws

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The Data

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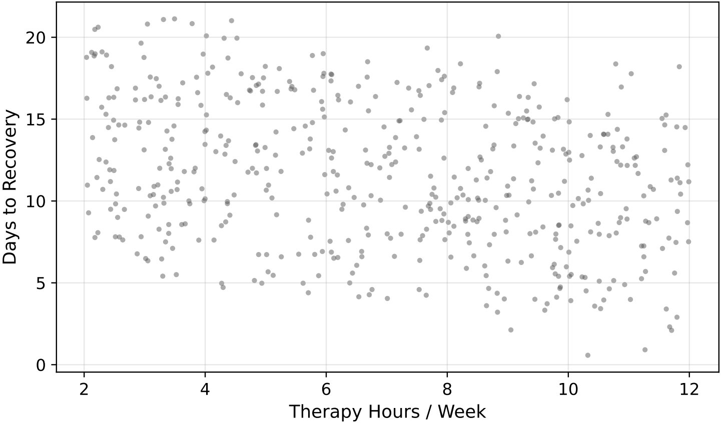

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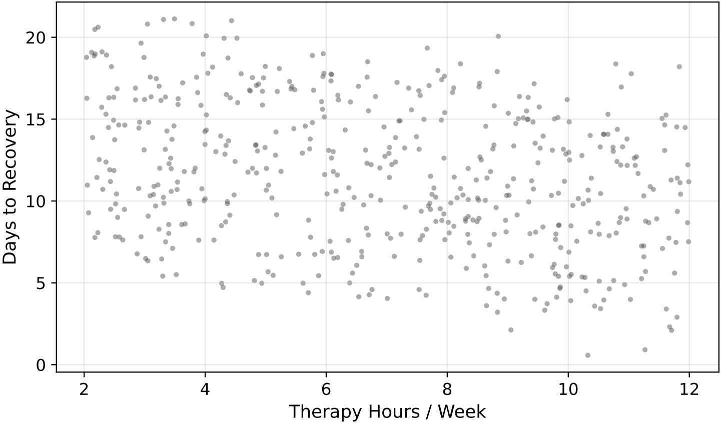
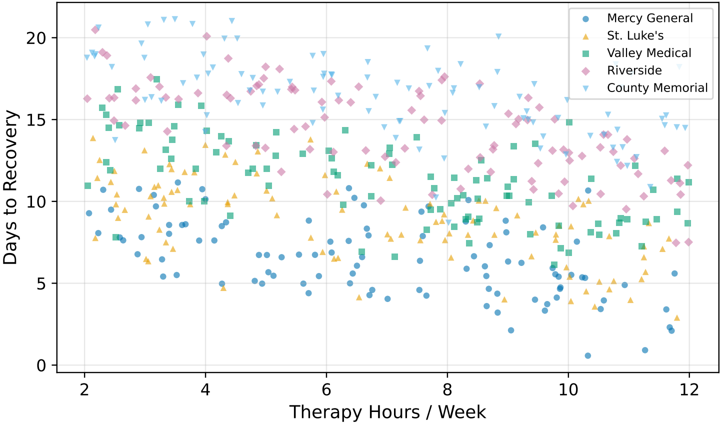


How could this data be generated?

Reveal: Five Hospitals

Patients come from **5 hospitals**, each with a different baseline recovery time.



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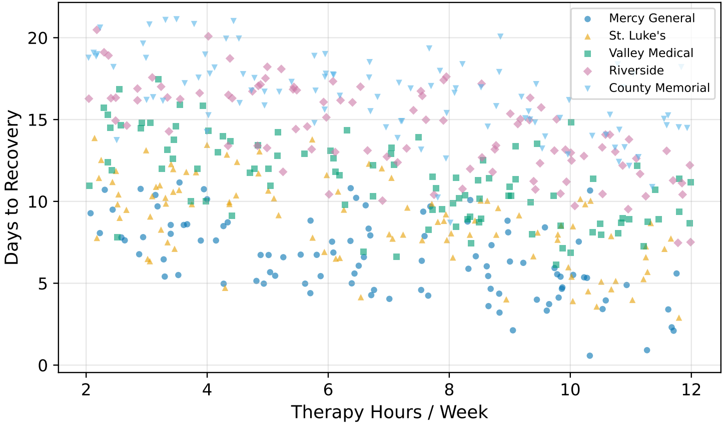

Recall: ignoring groups biases OLS. FE solved this by giving each group its own intercept.

FE Recap: Group-Specific Intercepts

The **fixed effects** model treats each hospital's baseline as a fixed unknown:

$$y_{ij} = \alpha_j + \beta x_{ij} + \varepsilon_{ij}$$

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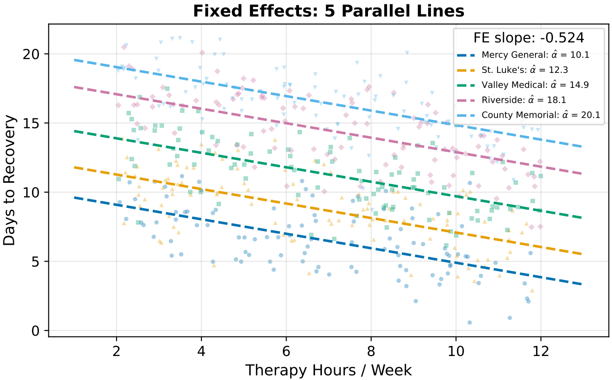
FE estimates a **separate intercept** for each hospital:



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$\Longrightarrow$ What if we could model the group effects with **fewer parameters**?

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Fixed effects: Each α_j is a fixed, unknown parameter to estimate.

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- $\bar{\alpha}$ = average baseline across *all* hospitals
- u_j = hospital j's **deviation** from that average (same role as α_j, but a random draw instead of a free parameter)
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Instead of estimating 5 (or 500) separate α_j's, we estimate $\bar{\alpha}$ and **one variance** σ_u^2.

The critical assumption:

$$\boxed{\text{Cov}(u_j, x_{ij}) = 0}$$

i.e., patients at a **better** (worse) hospital can't tend to receive **more** (less) therapy.

The Error Components Model

Substitute $\alpha_j = \bar{\alpha} + u_j$ into the FE model:

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Two patients at the same hospital share u_j, so their errors are **correlated**. OLS ignores this.

The Correlation Structure

Within hospital j (patients $i \neq k$):

$$\text{Corr}(v_{ij},\, v_{kj}) = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} > 0$$

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$\Longrightarrow$ Even if the OLS slope is OK, the inference is wrong.

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$\implies$ A hospital with only 5 patients gets a better estimate by "learning" from the other 495. The more hospitals look alike (small ρ), the more we can borrow.

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Then run OLS on the transformed data. This is **feasible GLS** (FGLS).

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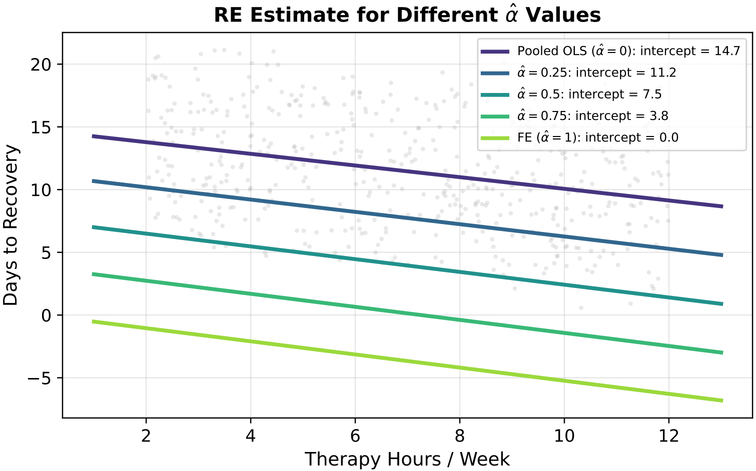
$\Longrightarrow$ With strong group effects or large panels, RE $\approx$ FE.

The $\hat{\alpha}$ Spectrum: Visualized



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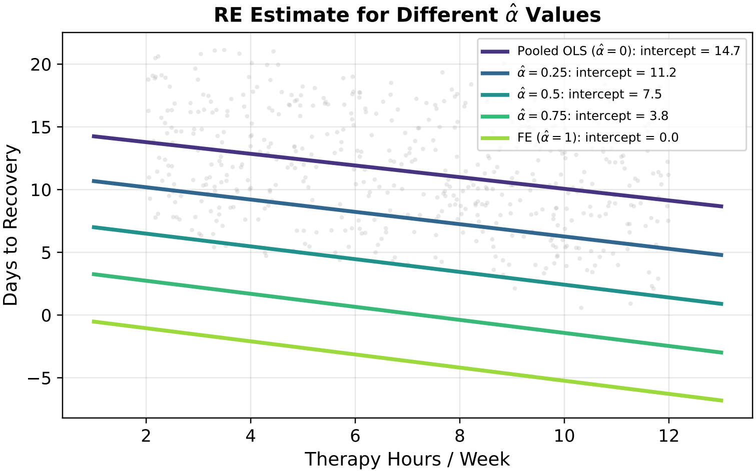

As $\hat{\alpha}$ increases from 0 to 1, we demean more aggressively, trusting each hospital's own data rather than pulling it toward the overall mean.

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If $\text{Cov}(u_j, x_{ij}) \neq 0$:

- FE is still consistent (it eliminates u_j entirely)
- RE is **inconsistent** (the partial demeaning doesn't fully remove u_j)

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4. **Efficiency**: RE uses both within and between variation
   → Smaller standard errors than FE

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$\Longrightarrow$ When in doubt, FE is the conservative choice. But how do we decide formally?

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If RE assumption holds: Both FE and RE are consistent, but RE is more efficient.

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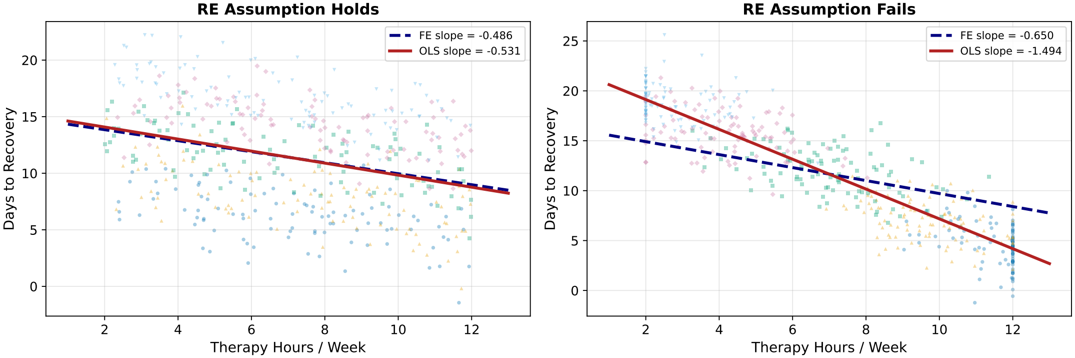

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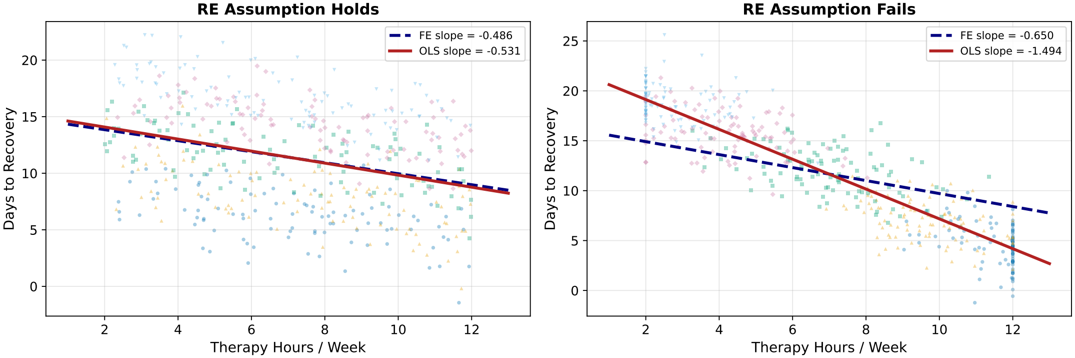

When the assumption holds, slopes are similar. When it fails, they **diverge**.

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With multiple regressors, the test generalizes to a χ^2_K statistic (where $K =$ number of regressors). Software handles this automatically.

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Why is the denominator well-defined?

Under H_0, FE is less efficient than RE:

$$\text{Var}(\hat{\beta}_{FE}) > \text{Var}(\hat{\beta}_{RE})$$

so $\widehat{\text{Var}}(\hat{\beta}_{FE}) - \widehat{\text{Var}}(\hat{\beta}_{RE}) > 0$ and the square root exists.

Interpreting the Hausman Test

Decision rule:

Result	**Conclusion**	**Action**
$p < 0.05$	Reject H_0	Use **FE**
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Intuition:

- Reject → FE and RE give **different** answers → RE is biased → use FE
- Fail to reject → FE and RE give **similar** answers → RE is OK and more efficient

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- Fail to reject $\rightarrow$ FE and RE give **similar** answers $\rightarrow$ RE is OK and more efficient

$\Longrightarrow$ The Hausman test is a **specification test**: it checks whether $\text{Cov}(u_j, x_{ij}) = 0$ is reasonable.

Worked Example: Hospital Data

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	FE	**RE**
Slope ($\hat{\beta}$)	-0.500	-0.497
$\widehat{\text{Var}}(\hat{\beta})$	0.00180	0.00165

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	FE	**RE**
Slope ($\hat{\beta}$)	-0.500	-0.497
$\widehat{\text{Var}}(\hat{\beta})$	0.00180	0.00165

Hausman statistic:

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Using our hospital recovery data:

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$\implies$ RE and FE agree. RE is appropriate here, and more efficient.

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$\Longrightarrow$ When in doubt, FE is the sa**fe** default. RE is the **re**ward for being able to argue $\text{Cov}(u_j, x_{ij}) = 0$.

Thank you!

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