LPM vs. Logit/Probit

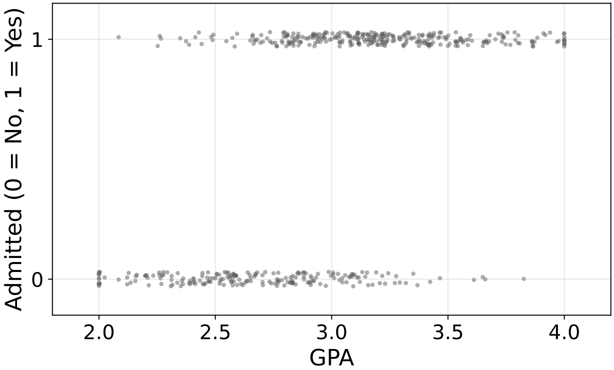
Jake Anderson

May 16, 2026

Outline

The Data

A university admissions office records **GPA** and **admission outcome** (admit/reject) for 500 applicants.



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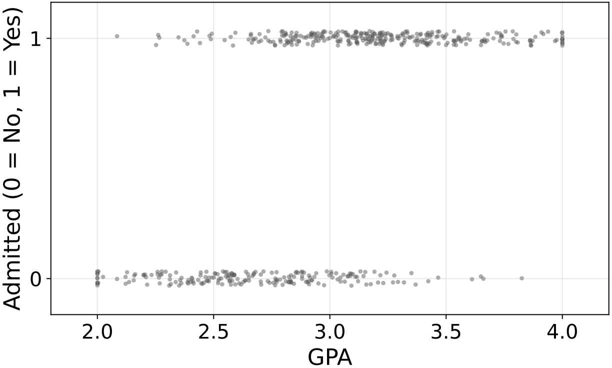

The outcome is binary: 0 (rejected) or 1 (admitted). How do we model the probability of admission?

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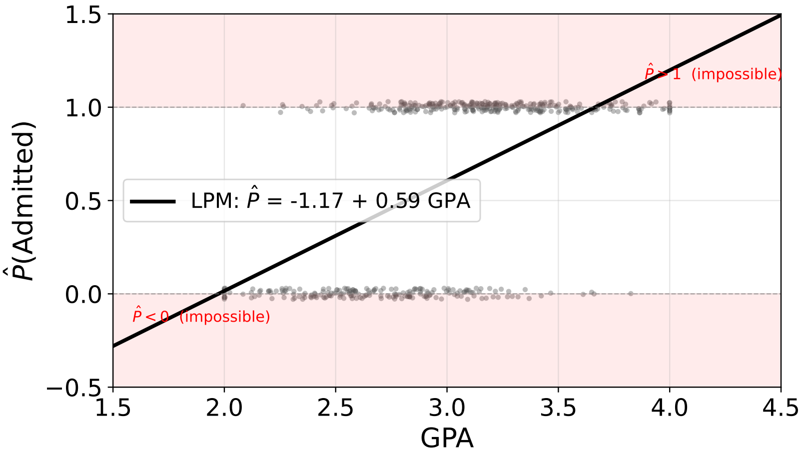
Sounds reasonable. Let's see what happens.

LPM Fit



1.5
1.0
0.5
0.0
−0.5
$\hat{P}$(Admitted)
$\hat{P} > 1$ (impossible)
LPM: $\hat{P}$ = -1.17 + 0.59 GPA
$\hat{P} < 0$ (impossible)
1.5
2.0
2.5
3.0
3.5
4.0
4.5
GPA

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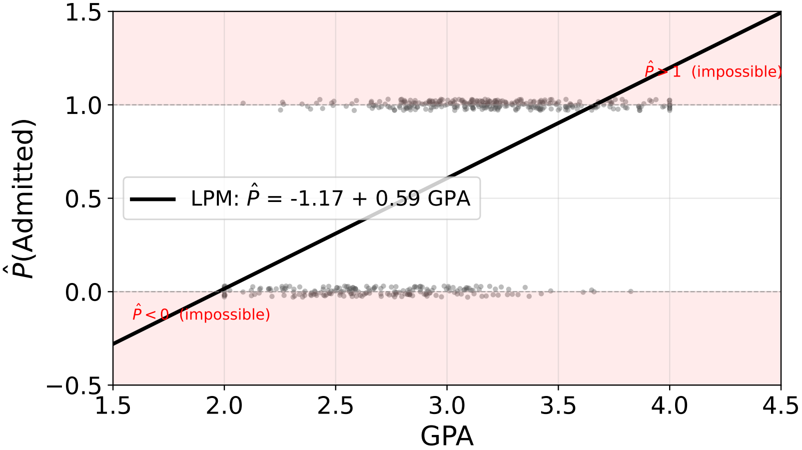

$\hat{P}(\text{Admit}) = -1.17 + 0.59 \cdot \text{GPA}$. At GPA $= 4.0$: $\hat{P} = 1.20$. At GPA $= 2.0$: $\hat{P} = 0.02$.

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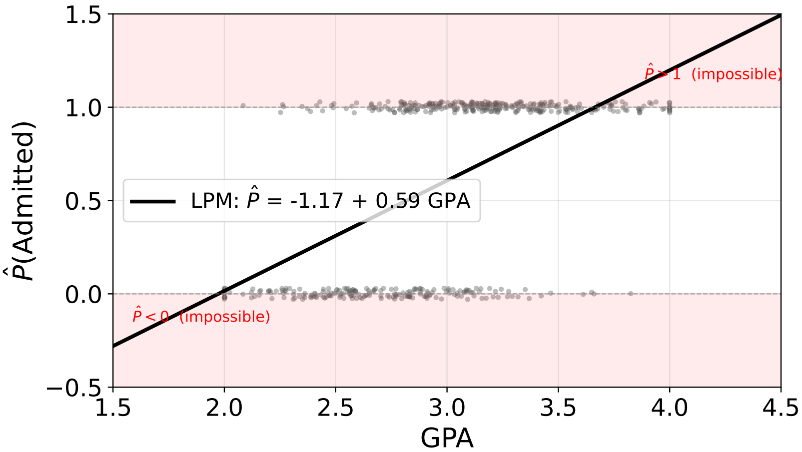

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$\implies$ The LPM is a line forced through inherently nonlinear data. It works in the middle but fails in the tails.

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$\Longrightarrow$ Marginal effects should be **largest near the midpoint** and diminish in the tails, not constant everywhere.

Problem 3: Heteroskedastic Errors

When y can only be 0 or 1, its variance is the variance of a Bernoulli random variable:

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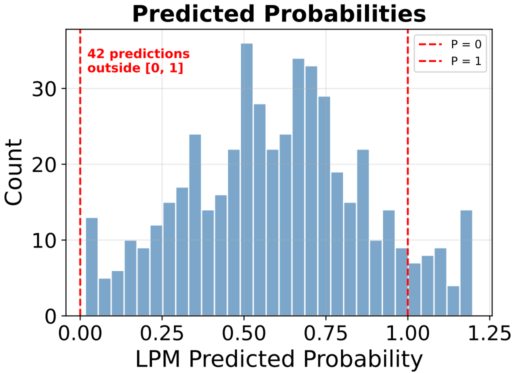
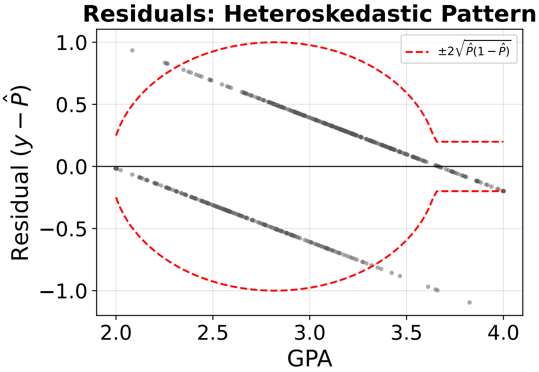
This problem is fixable: robust standard errors correct the SEs. But the impossible predictions and constant marginal effects remain.

LPM Problems: Visualized





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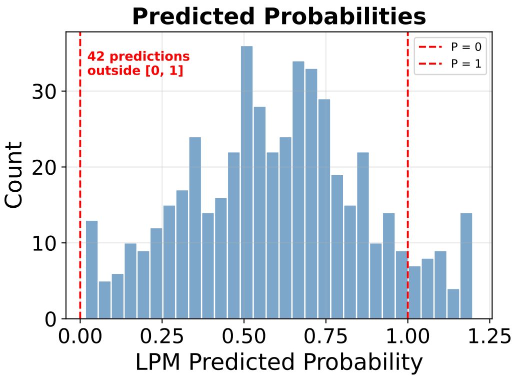

Left: some predictions fall outside $[0, 1]$. Right: residuals fan out, confirming heteroskedasticity.

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$\implies$ We need a **curve**, not a line: something that starts near 0, rises steeply through the middle, and flattens near 1.

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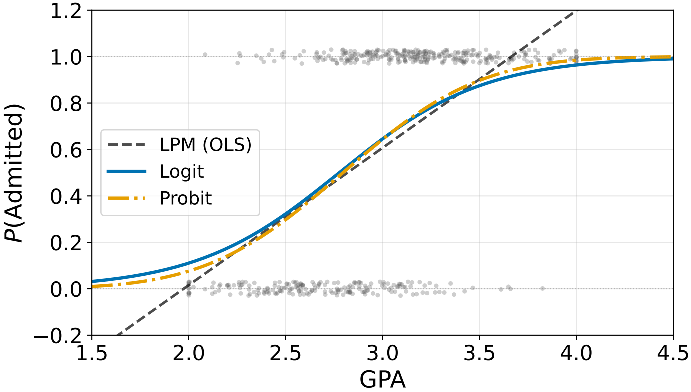

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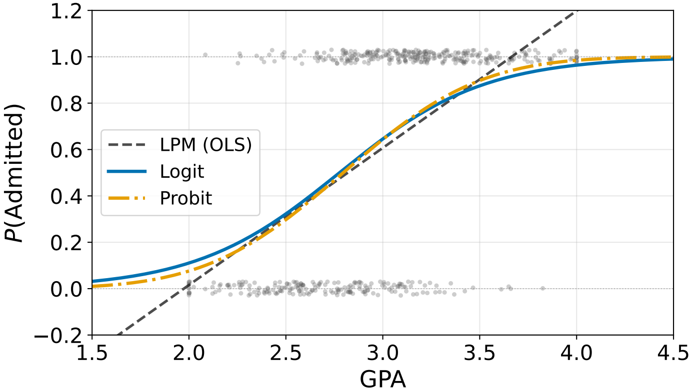

The LPM (dashed) overshoots at both ends. Logit and probit replace the line with an S-shaped curve that stays in $[0, 1]$.

Where Does the S-Curve Come From?

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$$\begin{aligned} P(\text{Admit} = 1 \mid \text{GPA}) &= P(y^* > 0) \\ &= P(\varepsilon > -\beta_0 - \beta_1 \, \text{GPA}) \end{aligned}$$

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$\implies$ The distribution we assume for ε determines the shape of the curve.

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$\implies$ Any CDF maps $(-\infty, +\infty) \to [0, 1]$, which is exactly what we need. Two standard choices give us two models.

Two Distributions, Two Models

Logistic distribution for ε:

$$P(\text{Admit} = 1 \mid \text{GPA}) = \Lambda(\beta_0 + \beta_1 \,\text{GPA}) = \frac{e^{\beta_0 + \beta_1 \,\text{GPA}}}{1 + e^{\beta_0 + \beta_1 \,\text{GPA}}}$$

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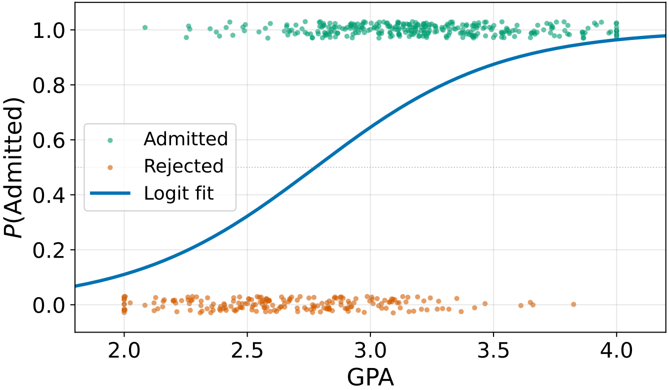
Both produce S-shaped curves bounded in $[0, 1]$. The logistic CDF (Λ) has slightly heavier tails than the normal CDF (Φ), but in practice the two are nearly indistinguishable.

The Logit Model: Fitted Curve



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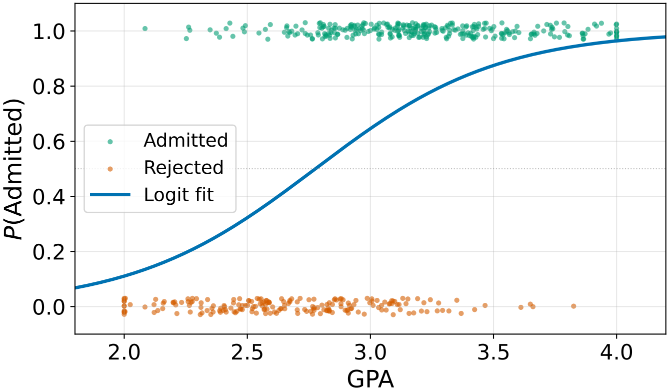

The logit curve passes through the middle of the data, stays in $[0, 1]$, and has the steepest slope near $P = 0.5$.

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$\implies$ To interpret logit coefficients, we need to understand what they actually measure.

Log-Odds: What the Coefficient Measures

Define the **odds** of admission:

$$\text{Odds} = \frac{P(\text{Admit} = 1)}{P(\text{Admit} = 0)} = \frac{P}{1 - P}$$

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Equivalently, the **odds ratio**:

$$e^{\beta_1} = e^{2.69} \approx 14.7$$

A one-unit increase in GPA **multiplies** the odds of admission by ≈ 14.7. For example, going from GPA 2.5 to 3.5 multiplies the odds by this factor.

Marginal Effects: What We Actually Want

The effect on *probability* depends on where you start:

$$\underbrace{\frac{\partial P}{\partial \,\text{GPA}}}_{\text{marginal effect}} = \beta_1 \cdot \Lambda(\beta_0 + \beta_1 \,\text{GPA}) \cdot \big(1 - \Lambda(\beta_0 + \beta_1 \,\text{GPA})\big)$$

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2.0	0.11	0.26
2.5	0.32	0.59
3.0	0.65	**0.61**
3.5	0.87	0.29
4.0	0.96	0.09

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$\Longrightarrow$ The same one-unit GPA increase has roughly 7x more impact near the middle than at the top.

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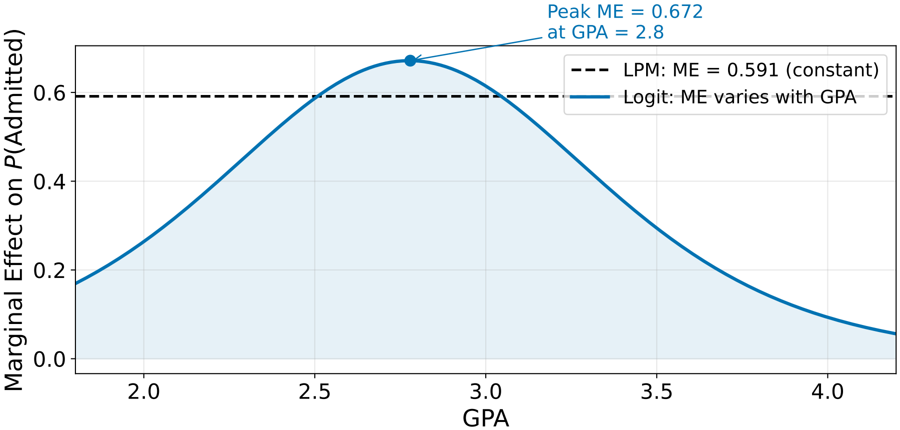

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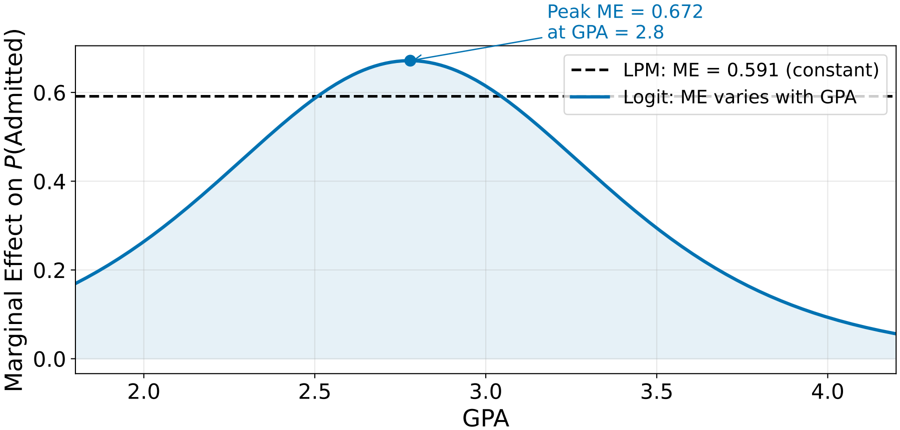

The LPM assumes a constant effect (dashed). The logit captures the realistic bell shape: largest effect near $P = 0.5$, vanishing in the tails.

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"On average, a one-unit increase in GPA is associated with a 49 percentage point increase in the probability of admission."

A full GPA point is a large change (e.g., 2.5 to 3.5), so this large AME makes sense in context.

Average Marginal Effect (AME)

Reporting a marginal effect at a single GPA is incomplete. Researchers typically report the **Average Marginal Effect**:

$$\text{AME} = \frac{1}{n}\sum_{i=1}^{n} \hat{\beta}_1 \cdot \hat{P}_i \cdot (1 - \hat{P}_i)$$

Compute the marginal effect *at each observation's actual GPA*, then average.

In our data: AME ≈ 0.49.

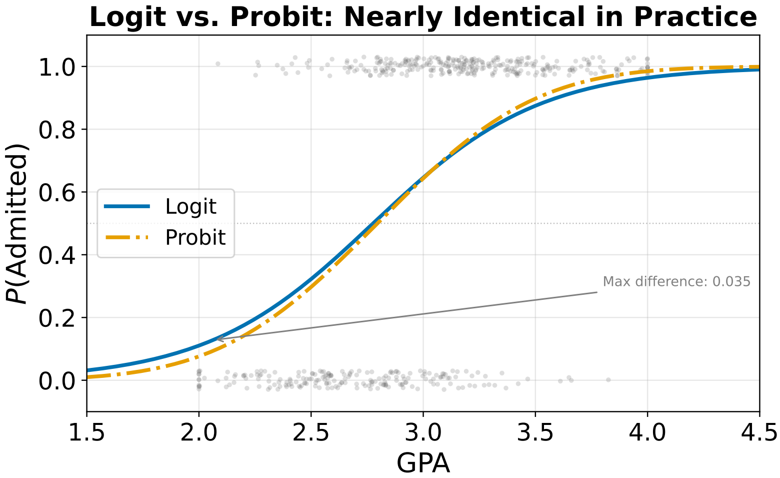
"On average, a one-unit increase in GPA is associated with a 49 percentage point increase in the probability of admission."

A full GPA point is a large change (e.g., 2.5 to 3.5), so this large AME makes sense in context.

$\Longrightarrow$ AME gives a single summary number comparable to the LPM coefficient (0.59). The LPM overstates the average effect because it ignores diminishing returns.

Outline

Logit vs. Probit: Nearly Identical



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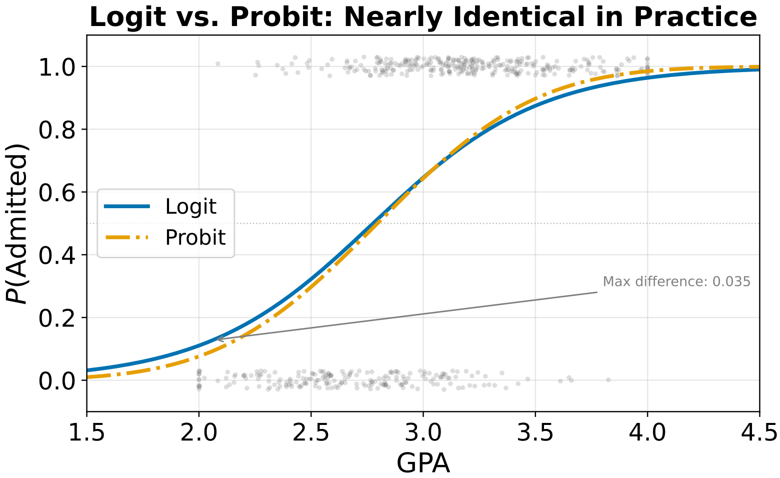

The two curves are almost indistinguishable. The largest difference is in the tails, where both curves are near 0 or 1.

Logit vs. Probit: Coefficients

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Three numbers you may see for the logit/probit coefficient ratio:

- $\sqrt{\pi^2/3} \approx 1.81$: the *theoretical* ratio, from the fact that the logistic distribution has variance $\pi^2/3$ while the standard normal has variance 1
- ≈ 1.6: a coarser textbook approximation (Amemiya)
- Here: $2.69/1.80 = 1.49$: the actual ratio in this finite sample

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$\Longrightarrow$ Marginal effects and predicted probabilities are nearly identical regardless. The choice between logit and probit rarely changes conclusions. Logit is more common in economics because of the odds-ratio interpretation.

Outline

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The LPM fails when:

1. You need predictions (e.g., credit scoring, medical diagnosis)
2. The outcome is rare or very common (P near 0 or 1)
3. You have covariates that push predictions far from 0.5

LPM vs. Logit: Decision Framework

	LPM	**Logit / Probit**
Estimation	OLS	MLE
Predictions in $[0, 1]$?	No	Yes
Marginal effects	Constant	Vary with x
Coefficient $=$ ME?	Yes	No (need AME)
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Speed / simplicity	Fastest	Slightly more complex
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Incidental parameters: with many FE, logit MLE estimates one parameter per group $\implies$ biased coefficients in short panels.

$\implies$ LPM + robust SEs is reasonable as a baseline; switch to logit/probit when predictions or nonlinear effects are central.

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- For an admitted applicant ($y_i = 1$): we want P_i to be **high**
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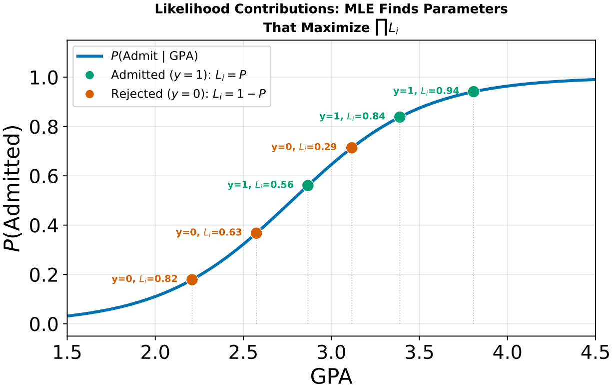
$\Longrightarrow$ MLE finds the S-curve that best separates the admitted from the rejected.

MLE: How It Works



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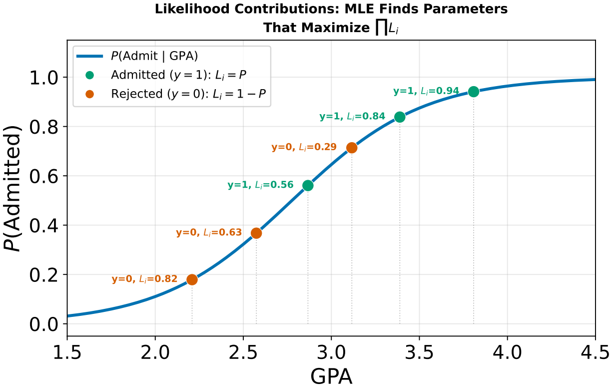

Each observation contributes P_i (if admitted) or $1 - P_i$ (if rejected) to the likelihood. MLE maximizes the product of these contributions.

The Log-Likelihood

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$\implies$ MLE is the standard estimation method for logit and probit. The resulting $\hat{\beta}$ values are the ones that maximize this log-likelihood.

Thank you!

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