The Heckman Selection Model

When Your Data Only Includes People Who Showed Up

Jake Anderson

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Outline

The Data: Wages and Education

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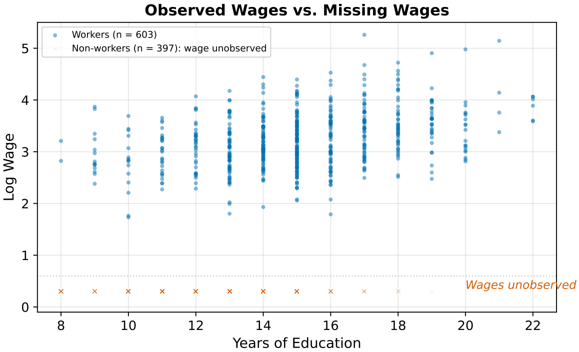

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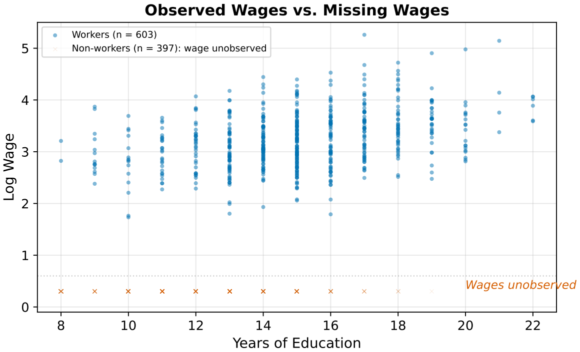

Blue dots: workers with observed wages. Orange crosses: non-workers with *no wage data*.

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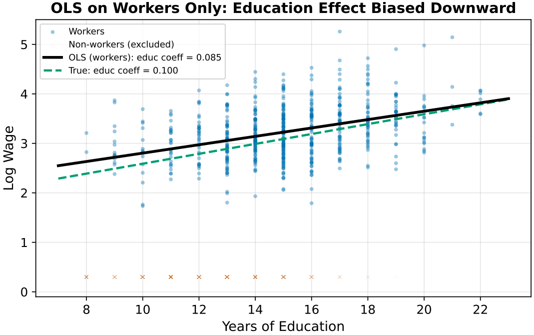
$\Longrightarrow$ This is **sample selection**: the decision to work is not random, and it correlates with the outcome we care about (wages).

OLS on Workers Only: Biased

Ignoring the missing data and running OLS on the 603 workers:

$$\widehat{\log(\text{wage})}_i = 1.373 + \underbrace{0.085}_{\substack{\text{biased downward} \\ \text{in this setting}}} \cdot \text{educ}_i + 0.040 \cdot \text{exper}_i$$



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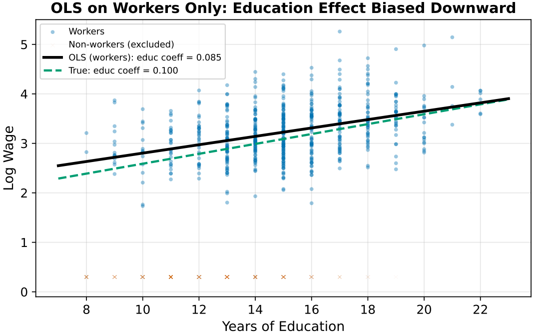
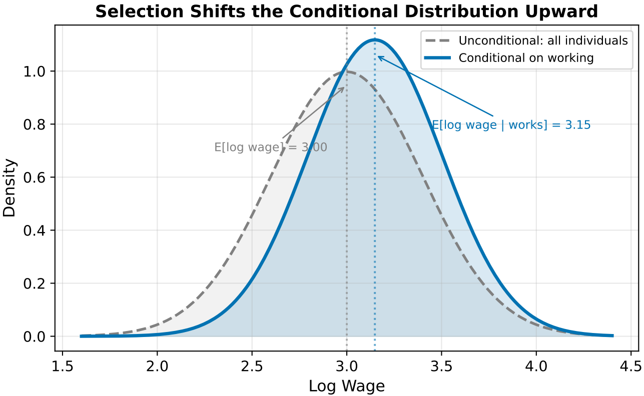


The OLS education coefficient is **0.085** (true = 0.100). That is a 15% underestimate. Why?

What OLS Misses: The Conditional Distribution



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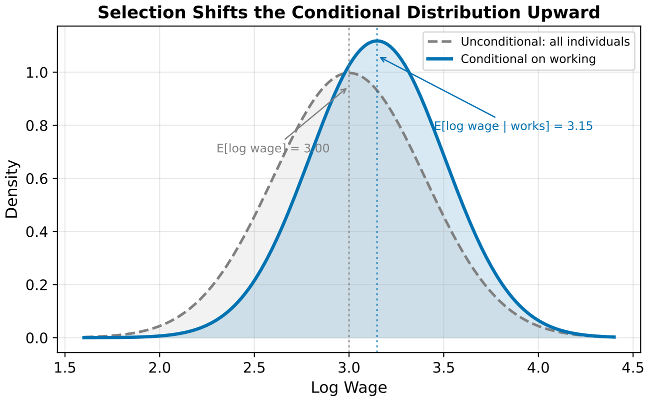

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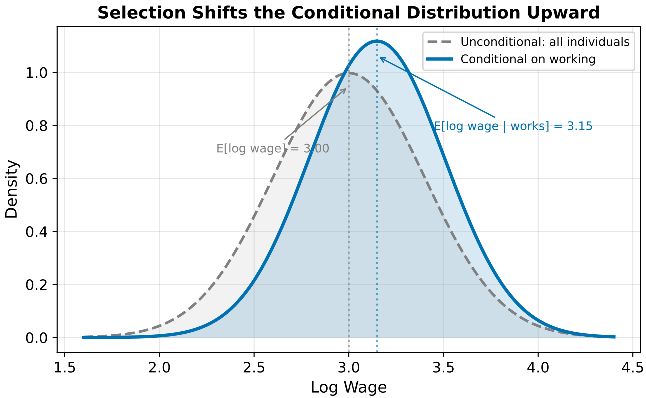

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OLS on workers fits a line through the blue distribution, but the true regression line passes through the gray one. The shift gets absorbed into the intercept and correlated coefficients, biasing them.

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- If you set missing wages to any fixed number, you change the distribution of the dependent variable in a way that depends on the selection process

$\Longrightarrow$ Neither dropping non-workers (OLS on workers) nor imputing values fixes the problem. We need a model that **explicitly accounts for the selection process**.

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$\Longrightarrow$ Our wage data has the same problem. Workers are not a random draw: they are the people whose unobserved characteristics (motivation, ability) pushed them into the labor force. These same characteristics also affect wages.

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3. **Recover the true return to education:** the structural relationship between education and wages, free from selection bias

$\Longrightarrow$ We need to jointly model the wage process and the selection process. This is different from Tobit (censoring): here the **decision to work is a separate equation** from the wage itself.

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$\Longrightarrow$ If u_i and v_i are correlated ($\rho \neq 0$), workers are a **selected** subsample and OLS on workers is biased.

Where the Bias Comes From: Correlated Errors

The Two-Equation Framework

Wage Equation (Outcome)

$$\log(\text{wage}_i^*) = 1.0 + 0.10 \cdot \text{educ}_i + 0.04 \cdot \text{exper}_i + u_i$$

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$\text{Corr}(u_i, v_i) = \rho = 0.6$

Unobserved factors that raise wages also raise the probability of working.

Observation Rule

We observe $\log(\text{wage}_i)$ only if $\text{work}_i^* > 0$

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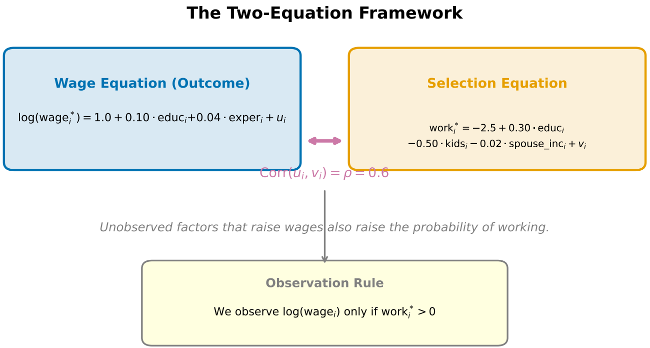

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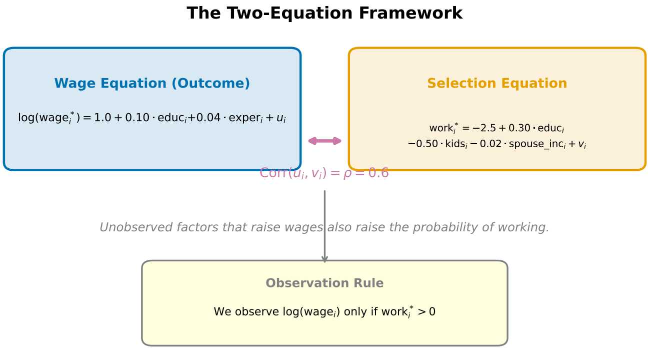

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$\implies$ OLS assumes $\text{E}[u_i \mid \text{works}] = 0$, but in reality $\text{E}[u_i \mid \text{works}] > 0$. This violates the zero conditional mean assumption.

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$\Longrightarrow$ If we can calculate this term and include it in our regression, we remove the bias.

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Standard normal PDF and CDF. Recall from your statistics courses:

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$\Longrightarrow$ In the probit model, $P(\text{work}_i = 1) = \Phi(\text{GZ}_i)$. Both ϕ and Φ will appear in the selection correction formula.

The Inverse Mills Ratio

For bivariate normal errors (u_i, v_i), the selection bias term has a closed form:

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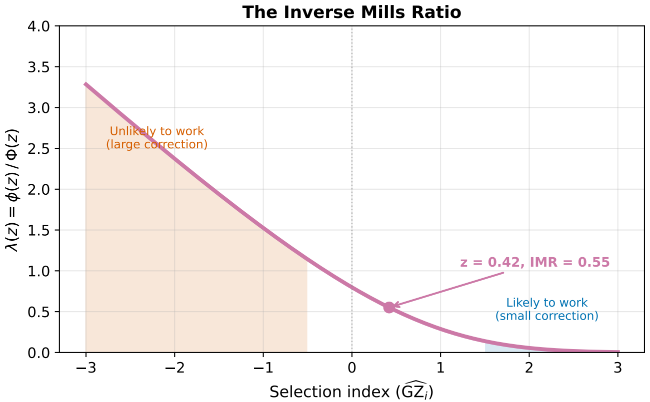
$\implies$ Connection to Tobit: you saw the IMR in the conditional expectation $E[y \mid y > 0]$. Same mathematical object, different context. In Tobit it corrects for censoring; here it corrects for selection.

How the IMR Works



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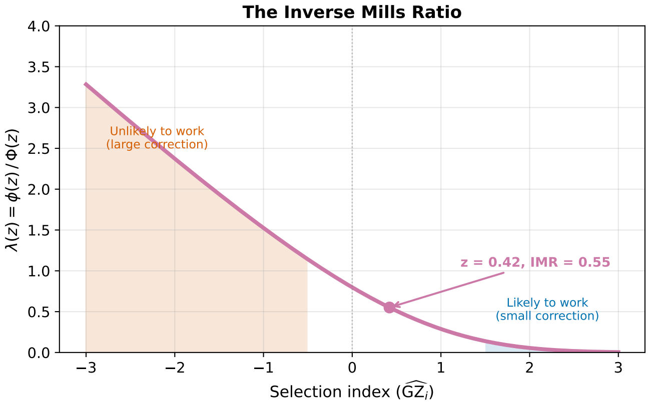

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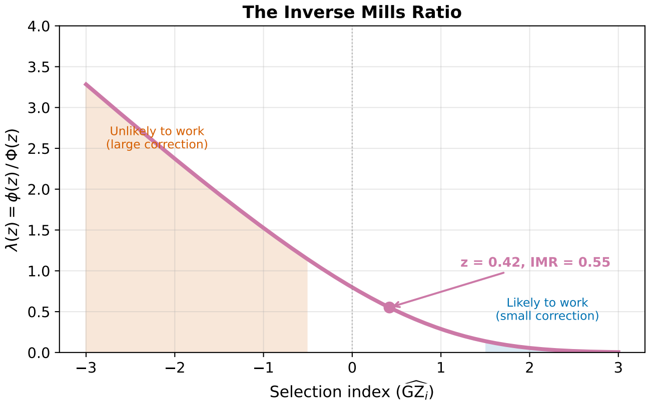

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When GZ_i is large (very likely to work), the IMR is **small**: working tells us little about her unobservables. The correction is minimal.

The Corrected Wage Equation

Substituting the IMR into the conditional expectation:

$$\mathrm{E}[\log(\text{wage}_i) \mid \text{works}] = \beta_0 + \beta_1 \, \text{educ}_i + \beta_2 \, \text{exper}_i + \underbrace{\rho\sigma_u}_{\equiv \delta} \cdot \lambda_i$$

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This is an OLS regression with one additional variable: λ_i. If we knew the γ coefficients (from the selection equation), we could compute λ_i for each worker and run this regression.

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This is an OLS regression with one additional variable: λ_i. If we knew the γ coefficients (from the selection equation), we could compute λ_i for each worker and run this regression.

$\Longrightarrow$ This is the logic behind the Heckman two-step procedure.

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$\Longrightarrow$ We need to estimate the γ coefficients first. This suggests a **two-step procedure**: (1) estimate the selection equation, (2) use the estimated $\hat{\lambda}_i$ in the wage regression.

Outline

Step 1: Estimate the Selection Equation (Probit)

Run a **probit** on all 1,000 observations (workers and non-workers):

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From these estimates, compute the estimated selection index $\widehat{\text{GZ}}_i$ for every individual.

Step 1 Continued: Compute the IMR

For each worker i, compute the inverse Mills ratio using the estimated selection index $\widehat{GZ}_i$:

$$\hat{\lambda}_i = \frac{\phi(\widehat{GZ}_i)}{\Phi(\widehat{GZ}_i)}$$

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Numeric example: a woman with 16 years of education, 2 children, spouse earning \$50k:

$$\begin{aligned}\widehat{\mathrm{GZ}} &= -2.629 + 0.314 \times 16 + (-0.512) \times 2 + (-0.019) \times 50 \\ &= -2.629 + 5.024 - 1.024 - 0.950 = 0.421\end{aligned}$$

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$\Longrightarrow$ This worker has a moderate selection correction. If she were nearly certain to work, $\hat{\lambda}$ would be close to zero.

Step 2: OLS with the IMR as an Extra Regressor

Run OLS on the **603 workers**, adding $\hat{\lambda}_i$ as an additional regressor:

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Parameter	OLS (workers)	Heckman	True
$\hat{\beta}_0$ (intercept)	1.373	0.874	1.0
$\hat{\beta}_1$ (education)	0.085	**0.111**	0.100
$\hat{\beta}_2$ (experience)	0.040	0.039	0.040
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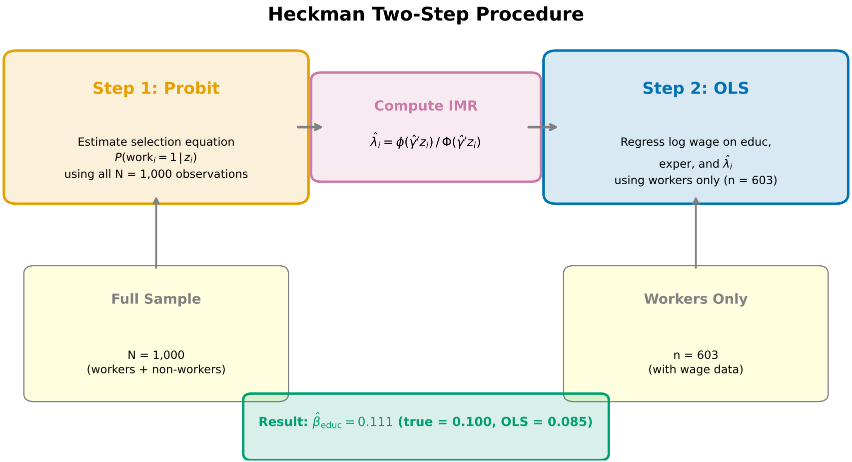
$\Longrightarrow$ The selection correction removes the downward bias in the education coefficient.

The Procedure in One Picture



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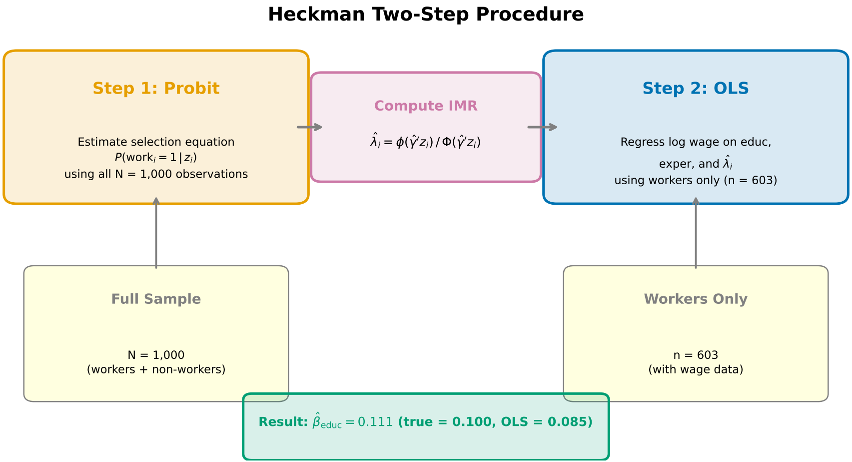
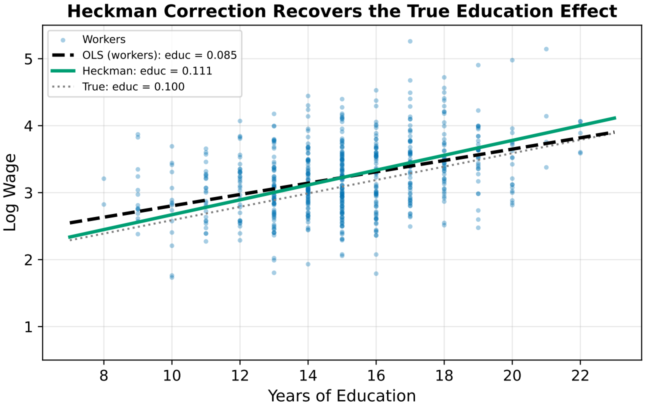


Step 1 uses **everyone** to learn who works. Step 2 uses **workers only** to learn how wages relate to education, after accounting for selection.

The Correction on the Scatter Plot



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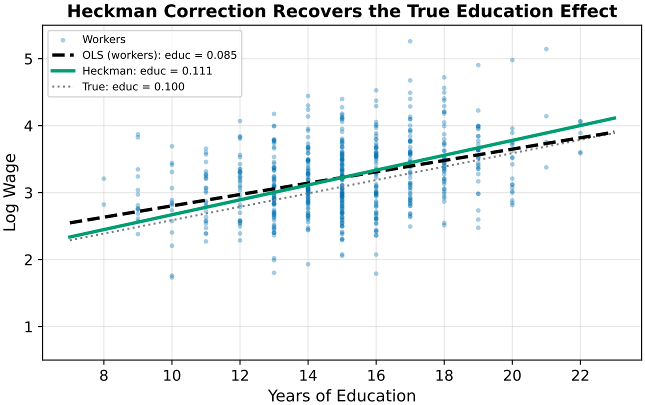

The dashed OLS line is too flat. The solid Heckman line is steeper, matching the true slope (gray dotted). Adding the IMR absorbs the selection effect that was biasing the education coefficient downward.

Interpreting the IMR Coefficient ($\hat{\delta}$)

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$\implies$ In our data, $\hat{\delta} = 0.260 > 0$: positive selection. The non-random sample of workers overrepresents high-ability individuals.

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What remains:

- What makes the Heckman model **credible**? (The exclusion restriction)
- How do we **test** whether selection bias is present?
- When does the model **fail**?

Outline

The Exclusion Restriction: Why Kids and Spouse Income

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$\Longrightarrow$ A credible exclusion restriction is what separates a convincing Heckman model from one that is essentially relying on the normality assumption.

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$\Longrightarrow$ In practice, many researchers run both and compare results. If they agree, the findings are more credible.

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$\implies$ In our data, $\hat{\delta} = 0.260$ is positive and statistically significant, confirming that selection bias is present and the correction is needed.

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$\Longrightarrow$ The Heckman model is powerful but not a magic fix. A credible exclusion restriction and reasonable normality are essential.

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     - Same mechanism for participation and amount? $\Longrightarrow$ **Tobit**
     - Different mechanisms? $\Longrightarrow$ **Two-Part Model**

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     - Same mechanism for participation and amount? $\Longrightarrow$ **Tobit**
     - Different mechanisms? $\Longrightarrow$ **Two-Part Model**
2. Is there no selection or censoring at all?
   - $\Longrightarrow$ **OLS** is fine

Decision Flowchart: Heckman vs. Tobit vs. OLS

1. Is your outcome missing for a non-random subset of observations?
   - **Yes:** the missing values come from a *separate selection process* (e.g., wages unobserved because the person does not work)
     - Do you have a valid exclusion restriction? $\Longrightarrow$ **Heckman Selection Model**
     - No exclusion restriction? $\Longrightarrow$ Consider **bounds** or **sensitivity analysis**
   - **No:** the zeros are *corner solutions* (the person would choose a negative value but is constrained)
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$\Longrightarrow$ Selection (Heckman) and censoring (Tobit) address different problems. The distinction is economic: does the outcome *exist but go unobserved*, or is it *constrained to a boundary*?

Outline

Summary: Back to Wages and Education

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5. **Testing:** if the IMR coefficient is significant, selection bias is present and the correction is needed
6. **Heckman vs. Tobit:** Tobit is for censoring (corner solutions). Heckman is for selection (missing data from a separate decision process)

James Heckman received the Nobel Prize in Economics in 2000, in part for developing this model.

Comparison: OLS vs. Heckman on Our Data

	OLS (workers)	**Heckman**	**True**
Education	0.085	0.111	0.100
Experience	0.040	0.039	0.040
IMR ($\hat{\delta}$)	–	0.260	0.240
Bias in educ	-15%	$+11\%$	–

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$\Longrightarrow$ The selection correction works because it accounts for the fact that workers are not a random sample. Ignoring selection systematically underestimates the return to education in this setting.

Thank you!

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